Compendium of Parameterized Problems

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This document includes a list of computational problems that have been studied within the framework of parameterized complexity [81]. It is mainly based on "A Compendium of Parameterized Complexity Results", version 2.0 (May 22, 1996), by Michael T. Hallett and H. Todd Wareham, and on Downey and Fellows' book [81]. It includes, however, several new results that have been published in the last few years.

Every computational problem in this document has one or more parameterized versions. Currently there are 312 computational problems listed in alphabetical order. For each of them we report the known parameterized versions (for a grand total of 376), the corresponding parameterized complexity results, and the references to the relevant articles in the bibliography.

This document does not pretend, of course, to be complete; for instance, it does not consider computational problems that are not decision problems. Moreover, this document likely includes errors and omissions. If you have corrections, suggestions, new or missing results, please send them to `compendium@sprg.uniroma2.it`.

Table of contents

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List of problems

Annotated Face Cover

Instance: A plane graph $G = (V, E)$ (that is, an embedding of a planar graph on the plane) with face set F; a function $\mu_V : V \to \{\text{active}, \text{marked}\}$; a function $\mu_F : F \to \{\text{active}, \text{marked}\}$; a positive integer k.

Question: Is there a set $C \subseteq \{f \in F \mid \mu_F(f) = \text{active}\}$ with $|C| \leq k$ and such that for each $v \in \{u \in V \mid \mu_V(u) = \text{active}\}$, there is a face in C whose boundary includes v?

Parameter: k

FPT ($O^*\big(4.6056^k\big)$ algorithm in [4, 118])

Note: See also Face Cover.

Arc Preserving Longest Common Subsequence

Instance: An alphabet Σ; sequences A, B over Σ such that $|A| = n$, $|B| = m$; arc sets $P_A \subset n \times n$, $P_B \subset m \times m$; a positive integer k.

Question: Is there an arc-preserving subsequence $\mathcal{S}$ of A and B of length at least k? A subsequence is considered *arc-preserving* if whenever both endpoints of an arc from either A or B are found in $\mathcal{S}$, the corresponding symbols are joined by an arc in the other sequence.

Parameter: k

W[1]-complete (hardness: reduction from Clique [99])

Note: The general problem is NP-complete by the reduction from Clique in [99]. The problem remains hard for both W[1] and NP even when arcs do not share endpoints.

Backward Paired Comparison

Instance: A digraph $D = (V, A)$; a weight function $\epsilon : V \to \mathbb{Q}$ such that the weights are proportional to $1/q$, where q is a positive integer, and moreover: (i) $\forall xy \in A$, $0 < \epsilon(xy) \leq 1$, (ii) $\forall xy, yx \in A$, $\epsilon(xy) + \epsilon(yx) = 1$, and (iii) $\forall xy \in A$ such that $yx \notin A$, $\epsilon(xy) = 1$; a nonnegative integer k.

Question: Does D have an ordering of backward length at most k? An *ordering* of D is a bijection $\alpha : V \to \{1, \ldots, |V|\}$. An arc $xy \in A$ is *backward* if $\alpha(x) > \alpha(y)$. The length of an arc $xy \in A$ is $\epsilon(xy) \cdot |\alpha(x) - \alpha(y)|$. The *backward length* of α is the sum of the lengths of all backward arcs.

Parameter: k

Open (reported in [141])

Parameter: k, q

FPT (proved in [141])

Parameter: cd(D)

FPT (proved in [141]; cd(D) represents the completion number of D)

Note: A weighted digraph satisfying the conditions shown above is said a *PCD* (*Paired Comparison Digraph*). The *completion number* of D is the minimal number of arcs that we have to add to D in order to obtain a semicomplete multipartite PCD. A digraph is *semicomplete multipartite* if it is obtained from a complete multipartite graph by replacing every edge (u, v) with the arc uv or the arc vu or both the arcs uv and vu.

α-Balanced Separator

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does there exist a set of vertices $S \subseteq V$ of cardinality at most k such that every connected component of $G[V \setminus S]$ has at most $\alpha\,|V|$ vertices? ($\alpha \in (0, 1)$ is a fixed constant)

Parameter: k

W[1]-hard, in W[P] (membership: reduction to Bounded Nondeterminism Turing Machine Computation [53]; hardness: reduction from Clique [35])

Bandwidth

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a 1:1 linear layout $f : V \to \{1, \ldots, |V|\}$ such that $(u, v) \in E$ implies $|f(u) - f(v)| \leq k$?

Parameter: k

W[t]-hard for all t (reduction from Uniform Emulation On A Path [28]; the problem remains W[t]-hard for all t when the given graph is directed and the layout must respect arc direction, or when the given graph is a tree [33])

Note: See also related problems Cutwidth, Linear Arrangement, and Linear Arrangement Generalized To A Vector p-Norm.

k-Based Tiling

Instance: A tiling systems with distinguished tiles; a positive integer k.

Question: Is there a tiling of the $n \times n$ plane using the tiling system and starting with exactly k distinguished tiles in a line?

Parameter: k

W[P]-complete (reduction to generic simulation of Turing machine [81, Exercise 13.0.4])

Binary Cladistic Character Compatibility

Instance: A set C of n binary cladistic characters over m objects; a positive integer k.

Question: Is there a subset $C' \subseteq C$, $|C'| = k$, such that all pairs of characters in C' are compatible?

Parameter: k

W[1]-complete (hardness: reduction from Clique [66, 210])

Note: The general problem is NP-complete by the reduction from Clique in [66]. The unconstrained-character version of this problem is also W[1]-complete [210]. If $k = |C|$, one obtains the Perfect Phylogeny problem.

Binary Qualitative Character Compatibility

Instance: A set C of n binary qualitative characters over m objects; a positive integer k.

Question: Is there a subset $C' \subseteq C$, $|C'| = k$, such that all pairs of characters in C' are compatible?

Parameter: k

W[1]-complete (hardness: reduction from Binary Cladistic Character Compatibility [66, 210])

Note: The general problem is NP-complete by the reduction from Binary Cladistic Character Compatibility in [66]. The unconstrained-character version of this problem is W[1]-hard [210].

Bipartite Colorful Neighborhood

Instance: A bipartite graph $G = (V_0, V_1, E)$; a positive integer k.

Question: Is there a two-coloring of V_1 such that there exists a set $S \subseteq V_0$ with $|S| \geq k$ such that each element of S has a colorful neighborhood, that is, each element of S has at least one neighbor of each color?

Parameter: k

FPT ($O^*\big(2.6494^k\big)$ algorithm using crown decomposition [163, 164])

Note: Equivalent to Set Splitting [163].

Bipartite Graph Embedding

Instance: A bipartite graph G; a bipartite graph H.

Question: Can H be embedded into G?

Parameter: H

W[1]-complete (membership: direct proof [81, Exercise 10.0.3]; hardness: reduction from Clique [38, 184], also in [81, Exercise 10.0.3])

Bipartite Matching Cardinality

Instance: A bipartite graph $B = (U, V, E)$; a positive integer k.

Question: Does G have at least k matchings?

Parameter: k

FPT (an algorithm is shown in [151])

Note: Equivalent to Permanent Lower Bound.

Bipartization By Edge Removal

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a set $C \subseteq E$ with $|C| \leq k$ whose removal produces a bipartite graph?

Parameter: k

FPT (by a parameterized reduction to Bipartization By Vertex Removal, which yields a $O^*\big(3^k\big)$ algorithm [213]; also [190])

Note: The parametric dual is Maximum Cut.

Bipartization By Edge Replacing With Length-2 Paths

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a set $C \subseteq E$ with $|C| \leq k$ such that replacing each edge in C by a path of length two produces a bipartite graph?

Parameter: k

FPT (equivalent to Bipartization By Edge Removal [118])

Bipartization By Vertex Removal

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a bipartization set $C \subseteq V$ with $|C| \leq k$ whose removal produces a bipartite graph?

Parameter: k

FPT (by a $O\big(3^k\,|V|\,|E|\big)$ algorithm in [190])

Bipartization Improvement

Instance: A graph $G = (V, E)$; a bipartization set $C \subseteq V$.

Question: Is there a bipartization set $C' \subseteq V$ with $|C'| < |C|$? A bipartization set is a subset of vertices whose removal produces a bipartite graph.

Parameter: $|C|$

FPT (by a $O\big(3^k\,|V|\,|E|\big)$ algorithm in [190])

Bounded Degree Red-Blue Nonblocker

Instance: A graph $G = (V, E)$ of maximum degree d ($d \geq 2$ is a fixed constant); a coloring of the vertices $c : V \to \{\text{red}, \text{blue}\}$; a positive integer k.

Question: Is there a set of red vertices V' of cardinality k such that every blue vertex has at least one neighbor that does not belong to V'?

Parameter: k

W[1]-complete (direct proof for both membership and hardness [78, 81])

Bounded DFA Intersection

Instance: An input alphabet Σ; a set of k deterministic finite-state automata $A_1, \ldots, A_k$ on the common input alphabet Σ; a positive integer m (let also define $q = \max\{|Q_i| : 1 \leq i \leq k\}$, where Q_i ($1 \leq i \leq k$) is the state set of the automaton A_i).

Question: Is there a string $X \in \Sigma^m$ that is accepted by each A_i, $i = 1, \ldots, k$?

Parameter: k

W[t]-hard for all t (reduction from Longest Common Subsequence parameterized by k [212]; it remains W[t]-hard for all t when the alphabet size $|\Sigma|$ is equal to 2 [212])

Parameter: m, q

W[2]-hard, in W[P] (membership: reduction to Bounded Nondeterminism Turing Machine Computation [53], hardness: reduction from Dominating Set [212]; still in W[2] if q is not a parameter [53])

Parameter: q

W[2]-hard (reduction from Dominating Set [212])

Parameter: k, m

W[1]-complete (membership: reduction to Short Nondeterministic Turing Machine Computation [53]; hardness: reduction from Longest Common Subsequence parameterized by k and m [212])

Parameter: m, $|\Sigma|$

FPT (From the $O(mk\,|\Sigma|^m)$ trivial algorithm that checks all $|\Sigma|^m$ possible strings [212])

Parameter: k, m, q

FPT (From a $O\left(m\,|\Sigma|^2\,q^{k+1}\right)$ algorithm that constructs the intersection DFA of all automata and then applies depth-first search to its transition diagram [212])

Parameter: k, q

Open (Reported in [212]; still open even when $|\Sigma|$ is also a parameter)

Parameter: q, $|\Sigma|$

Open (Reported in [212]; notice that a restricted version of the problem where $m \leq q$ is also in FPT when parameterized by q and $|\Sigma|$ from a $O\Big(mk^2\,|\Sigma|^2\,q^{(q^{2q|\Sigma|})+1}\Big)$ algorithm in [212], but in the general case a running time linear in m is not polynomial in the size of the instance)

Note: The unparameterized version of this problem where the string is in Σ^* is PSPACE-complete by a reduction from LINEAR SPACE ACCEPTANCE; the unparameterized version of this problem where the string is in Σ^m is NP-hard. The problem parameterized by $|\Sigma|$ is not in XP unless P = NP [212].

BOUNDED FACTOR FACTORIZATION

Instance: An n-bit positive integer N; a positive integer k.

Question: Is there a prime factor p of N such that $p < n^k$?

Parameter: k

randomized FPT ([108, 109])

BOUNDED HAMMING WEIGHT DISCRETE LOGARITHM

Instance: An n-bit prime p; a generator g of F_p^*; an element $a \in F_p^*$; a positive integer k.

Question: Is there a positive integer x whose binary representation has at most k 1's (that is, has a Hamming weight of no more than k) such that $a = g^x$?

Parameter: k

Open (reported in [81])

BOUNDED NONDETERMINISM TURING MACHINE COMPUTATION

Instance: A single-tape, single-head nondeterministic Turing machine $T = (\Sigma, Q, \Delta)$; an input word $x \in \Sigma^*$; positive integers k and m (m encoded in unary).

Question: Does there exist an accepting computation path of $T(x)$ having at most m steps and at most k nondeterministic steps?

Parameter: k

W[P]-complete (membership: direct proof [53]; hardness: reduction from CHAIN REACTION CLOSURE [53])

Note: This problem reduces to SHORT NONDETERMINISTIC TURING MACHINE COMPUTATION when both m and k are parameters.

BOUNDED WEIGHT t-NORMALIZED SATISFIABILITY

Instance: A t-normalized boolean expression X ($t \geq 2$); a positive integer k.

Question: Does X have a satisfying truth assignment of weight no more than k?

Parameter: k

W[t]-complete (direct proofs [44, 43, 45, 46]; W[t]-completeness also follows by the main lemma of Downey and Fellows [77]; in FPT for $t = 1$)

Note: A boolean expression is *t-normalized* if it is of the form product-of-sums-of-products ... of literals with t alternations.

CALL CONTROL IN TREE NETWORKS

Instance: A tree $T = (V, E)$; a *capacity* function $c : E \to \mathbb{N}$; a subset $R = \{(x_1, y_1), \ldots, (x_n, y_n)\} \subseteq V \times V$ of *connection requests*; an integer $k \geq 0$.

Question: Is there a subset $A \subseteq R$ such that $|R \setminus A| \leq k$ and no edge $e \in E$ is contained in more than $c(e)$ paths of P, where P is the set of paths of T that connect the pairs in A?

Parameter: k

FPT (from a $O^*\big(2^k\, k!\big)$ algorithm [197]; still in FPT with a $O^*(k!)$ algorithm for the directed graph obtained from a tree by replacing each edge with two directed arcs of opposite directions [197])

Note: See also CALL CONTROL IN TREES OF RINGS and CALL CONTROL IN GENERAL NETWORKS. If all edges have capacity equal to one, the problem is in P [176]. If all edges have capacity equal to one or two, there is a $O^*\big(3^\ell\big)$ algorithm, where ℓ is the number of edges of capacity equal to two [118].

CALL CONTROL IN TREES OF RINGS

Instance: A tree of rings $T = (V, E)$; a set $R = \{(x_1, y_1), \ldots, (x_n, y_n)\} \subseteq V \times V$ of *connection requests*; an integer $k \geq 0$.

Question: Is there a set $A \subseteq R$ such that $|R \setminus A| \leq k$ and there is a set of edge-disjoint paths in T that connect all pairs of A?

Parameter: k

FPT (from a $O^*(2.311^k)$ algorithm based on a reduction to HITTING SET FOR SIZE d SETS with $d = 3$ [197], then improved to a $O^*(2.1788^k)$ algorithm in [118]; still in FPT with a $O^*(5^k)$ algorithm for the directed graph obtained from a tree of rings by replacing each edge with two directed arcs of opposite directions [197])

Note: See also CALL CONTROL IN GENERAL NETWORKS and CALL CONTROL IN TREE NETWORKS. The classical, optimization version of CALL CONTROL IN TREES OF RINGS is NP-hard [98].

CALL CONTROL IN GENERAL NETWORKS

Instance: A graph $G = (V, E)$; a *capacity* function $c : E \to \mathbb{N}$ (let $C = \max\{c(e) : e \in E\}$); a subset $R = \{(x_1, y_1), \ldots, (x_n, y_n)\} \subseteq V \times V$ of *connection requests*; a set $P = \{p_i : p_i$ is a path of G that connects x_i and y_i, for every $(x_i, y_i) \in R\}$; a positive integer k.

Question: Is there a subset $A \subseteq R$ such that $|R \setminus A| \leq k$ and no edge $e \in E$ is contained in more than $c(e)$ paths of P?

Parameter: k

W[2]-hard, in W[P] (membership: reported in [197]; hardness: reduction from HITTING SET [197]; still W[2]-hard when restricted to series-parallel graphs [197])

Parameter: k, C

FPT (from a $O((C+1)^k\,|V|)$ algorithm in [197]; H. Fernau shows in [118] that this problem can be solved within the same time bounds as HITTING SET FOR SIZE d SETS with $d = C$)

Note: See also CALL CONTROL IN TREE NETWORKS and CALL CONTROL IN TREES OF RINGS. The CALL CONTROL problem without predetermined paths is NP-hard even for $k = 0$ on series-parallel graphs [176].

CAT AND MOUSE

Instance: A Cat and Mouse game $G = (X, E, c, M, v)$; a positive integer k.

Question: If $|M| = k$, does Player I have a winning strategy?

Parameter: k

XP-complete (membership: direct proof [81]; hardness: reduction from Pebble Game [5, 6])

Note: The Cat and Mouse Game is a quintuple $G = (X, E, c, M, v)$ with X a set of vertices, E a set of edges, $c \in X$, $M \subseteq X$, and $v \in X$. In the game, Player I begins with his token on c and Player II begins with tokens on each member of M. Players play alternatively and can move tokens from vertex x to vertex y provided $xy \in E$. Two tokens of Player II cannot occupy the same vertex. Player I wins if he can place his token on a vertex with one of Player II's tokens. Player II wins if she can place one of her tokens on vertex v even if it is occupied by Player I's token. Player I plays first.

Chain Minor Ordering

Instance: Two finite posets P and Q.

Question: Is P a chain minor of Q?

Parameter: P

Open (reported in [81])

Chain Reaction Closure

Instance: A directed graph $D = (V, A)$; a positive integer k.

Question: Does there exist a set V' of k vertices of D whose chain reaction closure is D? A *chain reaction closure* of V' is the smallest superset S of V' such that if $u, u' \in S$ and arcs $(u, x), (u', x) \in A$, then $x \in S$.

Parameter: k

W[P]-complete (hardness: reduction from Weighted Monotone Circuit Satisfiability [2])

Clique

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a set $V' \subseteq V$ of cardinality k such that $\forall\, u, v \in V'$, $(u, v) \in E$?

Parameter: k

W[1]-complete (equivalent to Independent Set [78])

Note: See also Planar Clique.

Clique Complement Cover

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does there exist a clique complement cover $C \subseteq V$ with $|C| \leq k$? A subset of vertices C is a *clique complement cover* if and only if $V - C$ induces a complete subgraph.

Parameter: k

FPT ($O^*\big(1.6182^k\big)$ algorithm in [118])

Note: This problem is strongly related to Vertex Cover, because C is a clique complement cover of G if and only if C is a vertex cover in the complement graph of G [118].

Clique For Almost Cluster Graphs

Instance: A graph $G = (V, E)$ that is a cluster graph with k edges added; positive integers k and s.

Question: Is there a clique of size s in G?

Parameter: k

FPT ($O\Big(1.53^k + |V|^3\Big)$ algorithm in [137])

Note: Strongly related to Cluster Deletion.

Cluster Deletion

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a edge set $C \subseteq E$ with $|C| \leq k$ whose removal produces a graph that consists of a disjoint union of cliques?

Parameter: k

FPT ($O\Big(1.53^k + |V|^3\Big)$ search tree algorithm [132, 133]; membership in FPT was established in [42])

Note: There exists some constant ϵ such that it is NP-hard to approximate this problem to within a factor of $1 + \epsilon$ [199]; see also Cluster Editing and Cluster Vertex Deletion.

Cluster Editing

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Can we transform G, by deleting and adding at most k edges, into a graph that consists of a disjoint union of cliques?

Parameter: k

FPT ($O\left(2.27^k + |V|^3\right)$ search tree algorithm [132, 133])

Note: The best known approximation factor for this problem is 4; the problem is APX-hard [56]; see also Cluster Deletion.

Cluster Vertex Deletion

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a vertex set $C \subseteq V$ with $|C| \leq k$ whose removal produces a graph that consists of a union of vertex-induced cliques?

Parameter: k

FPT ($O\left(2.1788^k + |V|^3\right)$ [118])

Note: See also Cluster Deletion.

3-CNF Satisfiability

Instance: A Boolean formula F in conjunctive normal form (CNF) with variables in X and each clause of at most three literals.

Question: Is there a satisfying assignment $\alpha : X \to \{\text{TRUE}, \text{FALSE}\}$ for F? (Let k be the number of clauses of size exactly three.)

Parameter: k

FPT ($O^*\left(1.6182^k\right)$ search tree algorithm in [118])

Parameter: $|X|$

FPT ($O\left(1.5045^{|X|}\right)$ algorithm in [157], based on a previous $O\left(1.619^{|X|}\right)$ algorithm in [169])

Parameter: $\lceil |X| \,/\, \log(|F|) \rceil$

M[1]-complete ([103, 104, 73])

CNF Satisfiability With Bounded Deficiency

Instance: A Boolean formula F in conjunctive normal form (CNF) with n variables and m clauses (in the following, d_F denotes the maximum deficiency of F).

Question: Is there a satisfying truth assignment for F?

Parameter: d_F

FPT ($O\left(2^{d_F} n^3\right)$ algorithm in [204]; the maximum deficiency of a formula is related to graph parameters, like treewidth, of graphs related to the formula, as discussed in [205, 204])

Note: The *deficiency* of a propositional formula in CNF with n variables and m clauses is equal to $m - n$; the *maximum deficiency* of F is the maximum deficiency over all subsets of F.

Cograph Vertex Deletion

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a vertex set $C \subseteq V$ with $|C| \leq k$ whose removal produces a cograph?

Parameter: k

FPT ($O\left(3.1150^k + |V|^3\right)$ algorithm in [133, 118])

Note: A *cograph* is (1) a graph having only one vertex, or (2) the union of two cographs, or (3) the complement of a cograph. A graph is a cograph if and only if it does not contain any set of four vertices that induce a path.

Colored Cutwidth

Instance: A graph $G = (V, E)$; an edge-coloring $c : E \to \{1, \ldots, r\}$; a positive integer k.

Question: Is there a 1:1 linear layout $f : V \to \{1, \ldots, |V|\}$ such that for each color $j \in \{1, \ldots, k\}$ and for each i, $1 \leq i \leq |V| - 1$, we have $|\{(u, v) \in E : c(u, v) = j \wedge f(u) \leq i \wedge f(v) \geq i + 1\}| \leq r$?

Parameter: k

W[t]-hard for all t (reduction from LONGEST COMMON SUBSEQUENCE parameterized by k [33, 34]; the directed version of this problem where $f(u) < f(v)$ if $(u,v) \in E$ is also hard for W[t], for all t [34]; the variant in which we refer to the size of $\{(u,v) \in E : c(u,v) = j \wedge f(u) < i \wedge f(v) \geq i+1\}$ is also W[t]-hard, for all t [34])

2-COLORED DIRECTED VERTEX SEPARATION NUMBER

Instance: Directed acyclic graph $G = (V, E)$, coloring $c : V \rightarrow \{1, 2\}$; positive integers k_1 and k_2.

Question: Is there a topological sort f of G such that for all i, $1 \leq i \leq n$, we have that $\text{width}_{1,f}(i) \leq k_1$ and $\text{width}_{2,f}(i) \leq k_2$? For $\zeta \in \{1, 2\}$, $\text{width}_{\zeta,f}(i)$ is the number of vertices v such that $c(v) = \zeta$, $f(v) \leq i$ and such that there is an arc $(v, v') \in E$ with $i \leq f(v')$.

Parameter: k_1, k_2

W[t]-hard for all t (reduction from a directed variant of bandwidth [22])

COLORED GRAPH AUTOMORPHISM

Instance: A $\{\text{red}, \text{blue}\}$-colored (bipartite) graph $G = (V, E)$; a positive integer k.

Question: Is there an automorphism preserving colors moving exactly k blue vertices?

Parameter: k

W[1]-hard (reduction from WEIGHTED ANTIMONOTONE q-CNF SATISFIABILITY with $q = 2$ [81, Exercise 9.0.2])

COLORED GRID HOMOMORPHISM

Instance: A square grid graph H whose vertices are colored with c colors ($c \geq 2$ is a fixed constant); a colored graph G.

Question: Is there a homomorphism from H to G? A *homomorphism* is a map $f : V(H) \rightarrow V(G)$ that is onto and that has the properties that (i) whenever $(x, y) \in E(H)$, then $(h(x), h(y)) \in E(G)$, and (ii) if x and y have the same color, then $h(x)$ and $h(y)$ have the same color.

Parameter: $|H|$

W[1]-complete (membership is easy; hardness by reduction from CLIQUE [136])

Note: Related to DATABASE CONJUNCTIVE QUERY EVALUATION. Solvable in polynomial time when $c = 1$.

COLORED PROPER INTERVAL GRAPH COMPLETION

Instance: A graph $G = (V, E)$; a vertex coloring $c : V \to \{1, \ldots, k\}$; a positive integer k.

Question: Does there exist a proper interval supergraph of G which respects c?

Parameter: k

W[1]-hard (reduction from INDEPENDENT SET [154])

Note: This problem is equivalent to COLORED UNIT INTERVAL GRAPH COMPLETION, as the class of unit interval graphs and proper interval graphs are equivalent.

COMPACT DETERMINISTIC TURING MACHINE COMPUTATION

Instance: A multi-tape, multi-head deterministic Turing machine M having t tapes and h heads per tape; a word x on the input alphabet Σ of M; a positive integer k.

Question: Does M on input x accept and visit at most k work tape squares?

Parameter: k

AW[SAT]-hard (reduction from QUANTIFIED BOOLEAN FORMULA SATISFIABILITY [50]; it remains AW[SAT]-hard when restricted to single-head and single-tape Turing machines [50], and also when restricted to single-head machines with empty input, exactly one non-final internal state, and binary alphabet [50]; the problem is not in XP (unless P = PSPACE) even for Turing machines having one head on each tape, empty input, one non-final internal state and binary alphabet [50]; the problem is in XP either when restricted to single-head machines with at most one writable tape and empty input, or when the number of tapes is parameter, or finally when the transition table is total, that is, for any global configuration there exists an applicable transition [50])

Parameter: k, t, h, $|\Sigma|$

FPT (from a $O\left(|\Sigma|^{(t+1)(5k+2h)}\, k^{5h(t+1)}\, |Q|^7\right)$ algorithm (Q being the state set of M) that builds a directed graph whose nodes represent the global configurations of M [50])

Compact Nondeterministic Turing Machine Computation

Instance: A multi-tape, multi-head nondeterministic Turing machine M having t tapes and h head per tape; a word x on the input alphabet Σ of M; a positive integer k.

Question: Is there an accepting computation of M on input x that visits at most k work tape squares?

Parameter: k

AW[P]-hard (reduction from Quantified Circuit Satisfiability [1, 2]; it remains AW[P]-hard when restricted to single-head and single-tape Turing machines [1, 2], and also when restricted to machines with empty input, one head per tape, exactly one non-final internal state, and binary alphabet [50]; the problem is not in XP (unless P = PSPACE) even for Turing machines having empty input, one non-final internal state and binary alphabet [50]; the problem is in XP either when restricted to machines with at most one writable tape and empty input, or when the number of tapes is parameter, or finally when the transition table is total, that is, for any global configuration there exists an applicable transition [50])

Parameter: k, t, h, Σ

FPT (from a $O\left(|\Sigma|^{(t+1)(5k+2h)}\, k^{5h(t+1)}\, |Q|^7\right)$ algorithm (Q being the state set of M) that builds a directed graph whose nodes represent the global configurations of M [50])

Connected Dominating Set

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a subset $D \subseteq V$ with $|D| \leq k$ such that D is both a connected set and a dominating set?

Parameter: k

W[2]-hard (equivalent to MINIMUM INNER NODE SPANNING TREE [118, 103])

CONSECUTIVE BLOCK MINIMIZATION

Instance: An $m \times n$ binary matrix M; a positive integer k.

Question: Is it possible to permute the columns of M to obtain a matrix M' that has at most k blocks of consecutive 1's? (A *block* of consecutive 1's is an interval of a row such that all entries in that interval are 1's.)

Parameter: k

FPT (by the reduction to problem kernel method [81, Exercise 3.2.6])

CONSTRAINED MINIMUM VERTEX COVER IN BIPARTITE GRAPHS

Instance: A bipartite graphs $G = (V_1, V_2, E)$; integers k_1, k_2.

Question: Is there a *minimum* vertex cover of G with at most k_1 vertices in V_1 and at most k_2 vertices in V_2?

Parameter: k_1, k_2

FPT (by a $O\left(2^{k_1+k_2} + |E|\sqrt{|V|}\right)$ algorithm in [145]; best current algorithm has time complexity in $O\left(1.26^{k_1+k_2} + (k_1+k_2)|V|\right)$, and it is based on the Dulmage-Mendelsohn decomposition for bipartite graphs [60])

Note: The problem is related to CONSTRAINT BIPARTITE VERTEX COVER. It is NP-hard by a reduction from CLIQUE [60].

CONSTRAINT BIPARTITE DOMINATING SET

Instance: A bipartite graph $G = (V_1, V_2, E)$; positive integer k_1, k_2.

Question: Is there a dominating set $D \subseteq V_1 \cup V_2$ with $|D \cap V_1| \leq k_1$ and $|D \cap V_2| \leq k_2$?

Parameter: k_1, k_2

W[2]-hard (reduction from RED-BLUE DOMINATING SET [118])

CONSTRAINT BIPARTITE VERTEX COVER

Instance: A bipartite graph $G = (V_1, V_2, E)$; positive integer k_1, k_2.

Question: Is there a vertex cover $C \subseteq V_1 \cup V_2$ with $|C \cap V_1| \leq k_1$ and $|C \cap V_2| \leq k_2$?

Parameter: k_1, k_2

FPT (by a $O\big(2^{k_1+k_2}k_1k_2 + (k_1+k_2)\,|G|\big)$ algorithm [165, 142]; the best current algorithm has time complexity in $O^*\big(1.3999^{k_1+k_2}\big)$ [120])

Note: Equivalent to Spare Allocation [159], and related to Constrained Minimum Vertex Cover in Bipartite Graphs. It is NP-hard by a reduction from Clique [159].

Crossing Number

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does G have an embedding in the plane with crossing number no more than k (that is, with at most k edges crossings)?

Parameter: k

Open (reported in [81])

Note: See also Crossing Number For Max Degree 3 Graphs. There seem to be some intricate problems with the definition of *crossing number*, see [177, 206, 118].

Crossing Number For Max Degree 3 Graphs

Instance: A graph $G = (V, E)$ all of whose vertices have maximum degree 3; a positive integer k.

Question: Does G have an embedding in the plane with crossing number no more than k (that is, with at most k edges crossings)?

Parameter: k

FPT ($O\Big(|V|^3\Big)$ algorithm for fixed k by the Robertson-Seymour Theorem [102, 192, 193] and also [81, page 444])

Note: See also Crossing Number.

Cutwidth

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is the cutwidth of G less than or equal to k? Or, equivalently, is there a one-to-one mapping $\sigma : V \to \{1, \ldots, |V|\}$ such that for every $1 \le i < |V|$, $|\{(u, v) \in E : \sigma(u) \le i < \sigma(v)\}| \le k$?

Parameter: k

FPT (Fellows and Langston [111, 115]; solvable in $O(f(k) \cdot |V|)$ [23])

Note: Given a layout $\sigma : V \to \{1, \ldots, |V|\}$ and $\alpha \in \mathbb{R}$, the value of the cut at α is the number of edges $(u, v) \in E$ with $\sigma(u) < \alpha$ and $\sigma(v) > \alpha$. The cutwidth of a layout is the maximum of the value of the cut over all α. The cutwidth of G is the minimum of the cutwidths of all possible layouts of G.

Database Conjunctive Query Evaluation

Instance: A database $d = \{D; R_1, \ldots, R_m\}$ where D is a domain and each R_i is a relation over D; a conjunctive query Q; a tuple t over D.

Question: Is t selected by the query Q over the database d?

Parameter: $|Q|$

W[1]-complete (membership: reduction to Weighted q-CNF Satisfiability [180]; hardness: reduction from Clique [180]. Assuming FPT $\neq$ W[1], the problem is in FPT if and only if the instance is restricted to conjunctive queries with underlying graphs of bounded tree-width; the underlying graph of a query has a vertex for each variable and an edge between two vertices if and only if there is an atom of the query including both corresponding variables [136])

Parameter: number of variables in Q

W[1]-hard, in W[2] (membership: reduction to Weighted CNF Satisfiability [180]; hardness: reduction from Clique [180]. The problem is W[1]-complete when the signature, i.e., the set of relations and their arity, is fixed [180].)

Note: A *query* is a function that maps the database d to a relation (of certain arity) over the same domain D; a *conjunctive query* corresponds to relational algebra with selection, projection, join and renaming. See also Database Datalog Query Evaluation, Database First-Order Query Evaluation, and Database Positive Query Evaluation.

Database Datalog Query Evaluation

Instance: A database $d = \{D; R_1, \ldots, R_m\}$ where D is a domain and each R_i is a relation over D; a Datalog query Q; a tuple t over D.

Question: Is t selected by the query Q over the database d?

Parameter: $|Q|$

W[P]-hard (reduction from Weighted Monotone Circuit Satisfiability [180])

Parameter: number of variables in Q

W[P]-hard (reduction from Weighted Monotone Circuit Satisfiability [180])

Note: A *query* is a function that maps the database d to a relation (of certain arity) over the same domain D; a *Datalog query* corresponds to relational algebra with selection, projection, join, renaming, union and recursion. See also Database Conjunctive Query Evaluation, Database First-Order Query Evaluation, Database Positive Query Evaluation.

Database First-Order Query Evaluation

Instance: A database $d = \{D; R_1, \ldots, R_m\}$ where D is a domain and each R_i is a relation over D; a first-order query Q; a tuple t over D.

Question: Is t selected by the query Q over the database d?

Parameter: $|Q|$

W[t]-hard for all t (reduction from Weighted t-Normalized Satisfiability [180])

Parameter: number of variables in Q

W[P]-hard (reduction from Weighted Monotone Circuit Satisfiability [180])

Note: A *query* is a function that maps the database d to a relation (of certain arity) over the same domain D; a *first-order query* corresponds to relational algebra with selection, projection, join, renaming, union and set difference. See also Database Conjunctive Query Evaluation, Database Datalog Query Evaluation, and Database Positive Query Evaluation.

Database Monotone Query Nonemptiness

Instance: A database $d = \{D; R_1, \ldots, R_m\}$ where D is a domain and each R_i is a relation over D; a query Q that has only existential quantifications and no negations.

Question: Is the relation of the query Q over the database d nonempty?

Parameter: $|Q|$

W[1]-complete ([88])

Note: A *query* is a function that maps the database d to a relation (of certain arity) over the same domain D.

Database Positive Query Evaluation

Instance: A database $d = \{D; R_1, \ldots, R_m\}$ where D is a domain and each R_i is a relation over D; a positive query Q; a tuple t over D.

Question: Is t selected by the query Q over the database d?

Parameter: $|Q|$

W[1]-complete (membership: reduction to Database Conjunctive Query Evaluation [180]; hardness: reduction from Clique [180].)

Parameter: number of variables in Q

W[SAT]-hard (reduction from Weighted Formula Satisfiability [180])

Note: A *query* is a function that maps the database d to a relation (of certain arity) over the same domain D; a *positive query* corresponds to relational algebra with selection, projection, join, renaming and union. See also Database Conjunctive Query Evaluation, Database Datalog Query Evaluation, and Database First-Order Query Evaluation.

Database Query Nonemptiness

Instance: A database $d = \{D; R_1, \ldots, R_m\}$ where D is a domain and each R_i is a relation over D; a query Q.

Question: Is the relation of the query Q over the database d nonempty?

Parameter: $|Q|$

AW[$*$]-complete ([88])

Note: A *query* is a function that maps the database d to a relation (of certain arity) over the same domain D.

Degree Three Subgraph Annihilator

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a set of k vertices $V' \subseteq V$ such that $G - V'$ has no subgraph of minimum degree 3?

Parameter: k

W[P]-complete (hardness: reduction from Weighted Monotone Circuit Satisfiability [2])

Digraph Kernel

Instance: A digraph $D = (V, A)$; a positive integer k.

Question: Does there exist a kernel in D of size at most k? A *kernel* is a set of nodes S such that S is independent and for every vertex $x \in V \setminus S$, there is $y \in S$ such that $xy \in A$.

Parameter: k

W[2]-hard (reduction from Independent Dominating Set [139])

Note: Not every digraph has a kernel. For instance, all odd length directed cycles have no kernels. See also Planar Digraph Kernel.

Diameter Improvement For Planar Graphs

Instance: A planar graph $G = (V, E)$; a positive integer k.

Question: Can G be augmented with additional edges in such a way that the resulting graph G' remains planar and the diameter of G' is at most k? (The *diameter* of a graph is the maximum distance between a pair of vertices.)

Parameter: k

FPT (application of the Robertson-Seymour Theorem [79, 193]; solvable in $O(|V|)$ time for fixed k [23])

Note: Is this problem the same as Planar Diameter Improvement in [81]? The question is whether there is a planar graph G' with $G \subseteq G'$ such that the diameter of G' is at most k. It is said to be in FPT by [70].

3-Dimensional Euclidean Generalized Mover

Instance: A set O of obstacle polyhedra; a set P of polyhedra that are freely linked together at a set of linkage vertices V such that P has k degrees of freedom of movement; initial and final positions p_I and p_F of P in 3-dimensional Euclidean space.

Question: Is there a legal movement of P from p_I to p_F? That is, is there a continuous sequence of translations and rotations of the polyhedra in P such that at each point in time no polyhedron in P intersects any polyhedron in O, and the polyhedra in P intersect themselves only at the linkage vertices in V?

Parameter: k

AW[SAT]-hard (reduction from COMPACT DETERMINISTIC TURING MACHINE COMPUTATION [55])

Note: The general version of this problem is PSPACE-complete by a reduction from LINEAR SPACE ACCEPTANCE.

DIRECTED FEEDBACK ARC SET

Instance: A directed graph $D = (V, A)$; a positive integer k.

Question: Is there a set I of k arcs such that each directed cycle of D contains a member of I?

Parameter: k

Open (reported in [81]; it has the same parameterized complexity of DIRECTED FEEDBACK VERTEX SET [141]. In FPT when restricted to *multipartite tournaments* (digraphs obtained from multipartite graphs by adding an orientation to every edge) [207, 141])

DIRECTED FEEDBACK VERTEX SET

Instance: A directed graph $D = (V, A)$; a positive integer k.

Question: Is there a set U of k nodes such that each directed cycle of D contains a member of U?

Parameter: k

Open (reported in [81]. Solvable in time $O\left(2^k |V|^2 (\lg |V| + k)\right)$ when restricted to *tournaments*, i.e., digraphs in which there is exactly one arc between every pair of distinct nodes [72]. Also in FPT when restricted to *multipartite tournaments* (digraphs obtained from multipartite graphs by adding an orientation to every edge) [188, 141])

Note: See also Directed Feedback Arc Set.

Directed Linear Arrangement

Instance: A directed graph $G = (V, A)$; a positive integer k.

Question: Is there a one-to-one mapping $\sigma : V \to \{1, \ldots, |V|\}$ that respects the orientation of G—that is, $\sigma(u) < \sigma(v)$ whenever $(u, v) \in A$—such that $\sum_{(u,v)\in A} (\sigma(v) - \sigma(u)) \leq k$?

Parameter: k

FPT ($O(2^k\,|G|)$ algorithm in [118])

Parameter: $k - |A|$

FPT ($O(2^k\,|G|)$ algorithm in [118])

Note: See also Linear Arrangement.

Directed Max Leaf

Instance: A digraph $D = (V, A)$; a nonnegative integer k.

Question: Does D contain a rooted tree with at least k leaves (nodes of out-degree equal to zero)?

Parameter: k

Open (suggested by Mike Fellows and reported in [141])

Note: A *rooted tree* is a digraph H obtained from a undirected tree T and from one of its vertices $x \in V(T)$ by orienting the edges of T in such a way that every path from x to another vertex y in T is a directed path from x to y in H.

Disjoint Paths

Instance: A graph $G = (V, E)$; $s_1, \ldots, s_k$ *start* vertices; $t_1, \ldots, t_k$ *end* vertices.

Question: Do there exist vertex disjoint paths $P_1, \ldots, P_k$ such that P_i starts at vertex s_i and ends at vertex t_i for $i = 1, \ldots, k$?

Parameter: k

FPT (Robertson and Seymour's $O(f(k) \cdot n^3)$ algorithm [192, 193])

Note: For planar graph, this problem can be solved in time $O(g(k) \cdot n)$ [189].

DISJOINT r-SUBSETS

Instance: A collection $\mathcal{F}$ of r subsets of a set X; a positive integer k.

Question: Are there k disjoint subsets in $\mathcal{F}$?

Parameter: r, k

FPT (by the perfect hashing method [81, Exercise 8.3.1])

DOMINATING CLIQUE

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a set of k vertices $V' \subseteq V$ that forms a complete subgraph of G and is also a dominating set of G?

Parameter: k

W[2]-complete (membership is trivial; hardness: reduction from DOMINATING SET [36]; the problem is in FPT if V' is also required to be *efficient*, that is, each vertex not in V' is dominated by exactly one vertex in V' [36])

DOMINATING REARRANGEMENT

Instance: A graph $G = (V, E)$; a subset $S \subseteq V$.

Question: Is there a dominating rearrangement $r : S \rightarrow N[S]$, with $r(s) \in N[s]$ for each $s \in S$, such that $r(S)$ is a dominating set for G?

Parameter: $|S|$

W[2]-complete (membership: reduction to SHORT MULTI-TAPE NONDETERMINISTIC TURING MACHINE COMPUTATION [118]; hardness: reduction from RED-BLUE DOMINATING SET [118])

DOMINATING SET

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a set of k vertices $V' \subseteq V$ with the property that every vertex of G either belongs to V' or has neighbor in V'?

Parameter: k

W[2]-complete (membership is trivial; hardness by a reduction from WEIGHTED CNF SATISFIABILITY [75, 77, 81])

Parameter: treewidth of G

FPT ($O\left(4^{\text{tw}(G)}\,|V|\right)$ dynamic programming algorithm in [11, 9])

Note: The problem is W[2]-hard if the dominating set is required to be either connected or *total* (for each vertex in V there is an edge to some vertex in V') [36]. The problem is W[1]-hard if a general, connected, or total dominating set is also required to be *efficient* (each vertex not in V' is dominated by exactly one vertex in V') [36]. See also PLANAR DOMINATING SET.

DOMINATING SET ON BOUNDED GENUS GRAPHS

Instance: A graph $G = (V, E)$ of bounded genus γ (γ constant); a positive integer k.

Question: Is there a set of k vertices $V' \subseteq V$ with the property that every vertex of G either belongs to V' or has neighbor in V'?

Parameter: k

FPT ($O\left((4\gamma + 40)^k n^2\right)$ search tree algorithm in [97])

Note: $\gamma = 1$ corresponds to PLANAR DOMINATING SET.

DOMINATING SET OF QUEENS

Instance: A $n \times n$ chessboard C; a positive integer k.

Question: Is it possible to place k queens on C such that all squares are dominated?

Parameter: k

FPT (by reduction to problem kernel of size $(2k + 1)^2$ coupled with a dynamic programming algorithm based on bounded treewidth, which yields a $O\left(15^{2k} + n\right)$ algorithm [118])

Parameter: $k - n/2$

Open (reported in [118])

DOMINATING THRESHOLD SET

Instance: A graph $G = (V, E)$; positive integer k and r.

Question: Is there a set of at most k vertices $V' \subseteq V$ such that for every vertex $u \in V$, $N[u]$ contains at least r elements of V'?

Parameter: k

W[2]-complete (membership: direct proof [80]; hardness: trivial reduction from Dominating Set)

(k, l)-Dominator

Instance: A digraph D; positive integers k and l.

Question: Does D contain a (k, l)-dominator?

Parameter: k, l

Open (introduced in [158] and reported in [141])

Note: A digraph $H = (V, A)$ is said a (k, l)-*dominator* if there exist partite sets U and W such that $|U| = k$, $|W| = l$, and $A = \{uw : u \in U, w \in W\}$.

Domino Treewidth

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is the domino treewidth of G at most k?

Parameter: k

W[t]-hard for all t (reduction from Longest Common Subsequence parameterized by k [30])

Dual of Coloring

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Can G be properly colored by $|V| - k$ colors?

Parameter: k

FPT (by the reduction to problem kernel method (Jan Telle) [81, Exercise 3.2.7])

Dual of Irredundant Set

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does G have an irredundat set of size $|V| - k$? (An *irredundant* *set* V' has the property that each vertex $u \in V'$ has a private neighbor, i.e., a vertex $u' \in N[u]$ such that $\forall\, v \in V' \setminus \{u\}$, $u' \notin N[v]$.)

Parameter: k

FPT (by the reduction to problem kernel method [81, Exercise 3.2.7])

Edge Average Min Linear Arrangement

Instance: A connected graph $G = (V, E)$; a positive integer k.

Question: Is there a one-to-one mapping $\sigma : V \to \{1, \ldots, |V|\}$ such that $\sum_{(u,v) \in E} |\sigma(u) - \sigma(v)| \leq k \cdot |E|$?

Parameter: k

para-NP-complete (the problem was introduced by Serna and Thilikos [198]; it is NP-complete for any $k \geq 2$ and is also para-NP-complete [140])

Note: The complexity class para-NP includes all parameterized problems with instances (I, k) that can be solved in time $O(f(k)\,|I|^c)$ by nondeterministic Turing machines [122, 124]. See also Linear Arrangement and Vertex Average Min Linear Arrangement.

Edge Dominating Set

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a subset $D \subseteq E$ with $|D| \leq k$ such that for each $e \in E$, either $e \in D$ or there exists $e' \in D$ that is incident on e?

Parameter: k

FPT ($O^*\big(2.6181^k\big)$ algorithm in [118])

Note: See also Edge Dominating Set.

Edge-Induced Clique

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a subset of edges $C \subseteq E$ with $|C| \geq k$ such that the subgraph induced by E is a clique?

Parameter: k

W[1]-complete (equivalent to CLIQUE [149, 118])

EDGE-INDUCED CLIQUE COMPLEMENT COVER

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does there exist an edge-induced clique complement cover $C \subseteq E$ with $|C| \leq k$? A subset of edges C is an *edge-induced clique complement cover* if and only if $E - C$ induces a complete subgraph.

Parameter: k

FPT ([118])

EXACT EVEN SET

Instance: A red/blue graph $G = (\mathcal{R}, \mathcal{B}, E)$; a positive integer k.

Question: Is there a set of k vertices $R \subseteq \mathcal{R}$ such that each member of $\mathcal{B}$ has an even number of neighbours in R?

Parameter: k

W[1]-hard (reduction from PERFECT CODE [89])

Note: See also EVEN SET, EXACT ODD SET, ODD SET.

EXACT ODD SET

Instance: A red/blue graph $G = (\mathcal{R}, \mathcal{B}, E)$; a positive integer k.

Question: Is there a set of k vertices $R \subseteq \mathcal{R}$ such that each member of $\mathcal{B}$ has an odd number of neighbours in R?

Parameter: k

W[1]-hard (reduction from PERFECT CODE [89])

Note: See also ODD SET, EXACT EVEN SET, EVEN SET.

EXACT CHEAP TOUR

Instance: A graph $G = (V, E)$; a weight function $w : E \rightarrow \mathbb{Z}$; positive integers S and k.

Question: Is there a tour through at least k nodes of G of cost exactly S?

Parameter: k

W[1]-hard (reduction from SUBSET SUM [78])

Note: See also SHORT CHEAP TOUR.

EXACT LONG CYCLE

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does G have a cycle of length exactly k?

Parameter: k

FPT (Downey and Fellows [81])

Note: See also LONG CYCLE, which requires a different proof technique.

EVEN SET

Instance: A red/blue graph $G = (\mathcal{R}, \mathcal{B}, E)$; a positive integer k.

Question: Is there a set of at most k vertices $R \subseteq \mathcal{R}$ such that each member of $\mathcal{B}$ has an even number of neighbours in R?

Parameter: k

Open (reported in [89])

Note: See also EXACT EVEN SET, EXACT ODD SET, ODD SET. Equivalent to the MINIMUM DISTANCE FOR LINEAR CODES problem.

FACE COVER

Instance: A plane graph $G = (V, E)$ (that is, an embedding of a planar graph on the plane) with face set F; a positive integer k.

Question: Is there a face cover set $C \subseteq F$ with $|C| \leq k$? A *face cover set* is a set of faces whose boundaries contain all vertices of G.

Parameter: k

FPT ($O^*\big(4.6056^k\big)$ algorithm in [4, 118]; H. Fernau reports that the base of the exponential can be improved to about 4.5 [118])

Note: Is this problem equivalent to PLANAR EMBEDDING FACE COVER? Bienstock and Monma also considered a variant of this problem where some preselected vertices need not to be covered [21]; the algorithms in [4, 118] solve this variant too. See also ANNOTATED FACE COVER.

Facility Location

Instance: A bipartite graph $G = (F, C, E)$, consisting of a set F of potential *facility locations*, a set C of *customers*, and an edge relation E, where $(f, c) \in E$ indicates that c can be served by the facility at f; weight functions $w_F : F \to \mathbb{N}$ and $w_E : E \to \mathbb{N}$; a positive integer k.

Question: Is there a set $F' \subseteq F$ of facility locations and a set $E' \subseteq E$ of ways to serve customers such that (1) E' covers no vertex in $F - F'$, (2) E' covers all vertices in C, and (3) $\sum_{f \in F'} w_F(f) + \sum_{e \in E'} w_E(e) \leq k$?

Parameter: k

FPT (by reduction to a problem kernel of size $\leq k^{3k+1}$ discovered by M. Fellows and H. Fernau, which yields a $O\big(k^{3k+1}2^k + nm\big)$ algorithm [118], or by a $O\big(4^k nm\big)$ algorithm without kernerlization [118])

Note: An alternative formulation of this problem is as follows: given a matrix $M \in \mathbb{N}^{(n+1)\times m}$, indexed as $M[0 \ldots n][1 \ldots m]$, and a positive integer k, is there a set $C \subseteq \{1, \ldots, m\}$ of columns and a function $s : \{1, \ldots, n\} \to C$ such that $\sum_{f \in C} \Big(M[0, f] + \sum_{c:\, s(c)=f} M[c, f]\Big) \leq k$? (Missing edges can be represented by weights in the matrix larger than k.)

Feasible Register Assignment

Instance: A directed acyclic graph $G = (V, E)$; a positive integer k; a register assignment $r : V \to \{R_1, \ldots, R_k\}$.

Question: Is there a linear ordering f of G, and a sequence $S_0, S_1, \ldots, S_{|V|}$ of subsets of V, such that $S_0 = \emptyset$, $S_{|V|}$ contains all vertices of in-degree 0 in G, and for all i, $1 \leq i \leq |V|$, $f^{-1}(i) \in S_i$, $S_i - \big\{f^{-1}(i)\big\} \subseteq S_{i-1}$, and S_{i-1} contains all vertices u for which $(f^{-1}(i), u) \in E$, and for all j, $1 \leq j \leq k$, there is at most one vertex $u \in S_i$ with $r(u) = R_j$?

Parameter: k

W[t]-hard for all t (reduction from Longest Common Subsequence parameterized by k [33])

Feature Set

Instance: A set of examples $X = \big\{x^{(1)}, \ldots, x^{(m)}\big\}$, where, for all $1 \leq i \leq m$, $x^{(i)} = \Big\{t^{(i)}, x_1^{(i)}, \ldots, x_n^{(i)}\Big\} \in \{0, 1\}^{n+1}$; a positive integer k.

Question: Does there exist a *feature set* $S \subseteq \{1, \ldots, n\}$ with $|S| = k$ and such that for all pairs of examples $i \neq j$, if $t^{(i)} \neq t^{(j)}$ then there exists $q \in S$ such that $x_q^{(i)} \neq x_q^{(j)}$?

Parameter: k

W[2]-complete (membership: direct proof [63]; hardness: reduction from Dominating Set [63])

Note: An example is composed of a binary value specifying the value of the *target feature* and a vector of n binary values specifying the values of the other features.

Feedback Vertex Set

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a set U of at most k vertices of G such that each cycle of G passes through some vertex of U?

Parameter: k

FPT (Downey and Fellows [79]; also Bodlaender's $O(f(k) \cdot n)$ algorithm [24]; recently, a $O^*(10.567^k)$ algorithm has been devised [69])

Note: See also Planar Feedback Vertex Set.

Fixed Alphabet Longest Common Subsequence

Instance: An alphabet Σ having fixed size; a set of k strings $r_1, \ldots, r_k$ over the alphabet Σ; a positive integer λ.

Question: Is there a string $s \in \Sigma^*$ of length at least λ that is a subsequence of each r_i, for $i = 1, \ldots, k$? (A string s is a *subsequence* of a string r if we can delete some characters in r such that the remaining string is equal to s.)

Parameter: k

W[1]-hard (reduction from Partitioned Clique [182])

Parameter: k, λ

FPT (by the trivial algorithm that generates all $|\Sigma|^\lambda$ possible subsequence strings and checks them against each r_i)

Note: See also Longest Common Subsequence.

FIXED ALPHABET SHORTEST COMMON SUPERSEQUENCE

Instance: An alphabet Σ having fixed size; a set of strings $\{r_1, \ldots, r_k\}$ formed over alphabet Σ; a positive integer λ.

Question: Does there exist a string $s \in \Sigma^*$ of length at most λ such that s is a supersequence of each string r_i, $1 \leq i \leq k$? (A string s is a *supersequence* of a string r if we can delete some characters in s such that the remaining string is equal to r.)

Parameter: k

W[1]-hard (reduction from PARTITIONED CLIQUE [182])

Parameter: λ

FPT (reported by [105])

Note: See also SHORTEST COMMON SUPERSEQUENCE.

GATE MATRIX LAYOUT

Instance: A boolean matrix M; a positive integer k.

Question: Is there a permutation of the columns of M so that if in each row we change to $*$ every 0 laying between the row's leftmost and rightmost 1's, then no column contains more than k 1's and $*$'s?

Parameter: k

FPT (Fellows and Langston [113]; also Bodlaender's $O(f(k) \cdot n)$ algorithm [23])

Note: Equivalent to PATHWIDTH.

GENERALIZED VERTEX COVER

Instance: A graph $G = (V, E)$; a subset of vertices $V' \subseteq V$; a positive integer k.

Question: Is there a vertex cover $C \subseteq V'$ with $|C| \leq k$?

Parameter: k

FPT (equivalent to VERTEX COVER [118])

GENUS k-COVER

Instance: A graph G; a positive integer k.

Question: Is there a finite graph H of genus at most k that is a cover of G? H is a *cover* of G if there is a projection map $p : V(H) \rightarrow V(G)$ such that: (1) p is onto, and (2) if $p(u) = x$ and $(x, y) \in E(G)$, then there is a unique vertex $v \in V(H)$ with $(u, v) \in E(H)$ and $p(v) = y$.

Parameter: k

nonuniform FPT (solvable in time $O\left(|V|^3\right)$ by means of the Robertson-Seymour Theorem [81, 193])

Graph Genus

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does G have genus k?

Parameter: k

FPT ($O\left(|V|^3\right)$ algorithm for fixed k by the Robertson-Seymour Theorem [112, 193]; solvable in time $O(f(k) \cdot |G|)$ [168])

Graph Linking Number

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Can G be embedded into the 3-dimensional space in such a way that the maximum size of a collection of topologically linked disjoint cycles is bounded by k?

Parameter: k

nonuniform FPT (Fellows and Langston's $O\left(|V|^3\right)$ algorithm for fixed k by the Robertson-Seymour Theorem [112, 193])

Graph Modification Problem

Instance: A graph $G = (V, E)$; non-negative integers i, j, k.

Question: Can we delete at most i vertices, j edges, and add at most k edges and get a Π graph? (Π is any nontrivial hereditary property characterized by a finite set of forbidden induced subgraphs.)

Parameter: i, j, k

FPT ([42])

GRAPH PACKING

Instance: Graphs G and H; a positive integer k.

Question: Are there at least k vertex disjoint instances of H in G?

Parameter: k, $|H|$

FPT ($2^{O(|H|k \log k + k|H| \log|H|)}$ algorithm in [106])

Note: See also 3-PATH PACKING, s-STAR PACKING, and TRIANGLE PACKING.

GRAPH PSEUDO HOMOMORPHISM

Instance: Graphs $G = (V, E)$ and $H = (U, A)$.

Question: Is there a pseudo-homomorphism from G to H? A *pseudo-homomorphism* is a map $f : V \to U$ that is onto and that has the property that whenever $(x, y) \in A$, then there is a vertex $u \in f^{-1}(x)$ and there is a vertex $v \in f^{-1}(y)$ such that $(u, v) \in E$.

Parameter: H

FPT (by the perfect hashing method [81, Exercise 8.3.4])

GROUPING BY SWAPPING

Instance: A finite alphabet Σ; a string $x \in \Sigma^*$; a positive integer k.

Question: Is there a sequence of k or fewer adjacent symbol interchanges that transforms x into a string x' in which all occurrences of each symbol $a \in \Sigma$ are in a single block?

Parameter: k

FPT (by the reduction to the problem kernel method [81, Exercise 3.2.4])

HALF-WEIGHT $(t + 1)$-NORM SAT$^{\log n}$

Instance: A t-normalized circuit C with n gates and $k \log n$ input lines.

Question: Does there exist a *half-weight* satisfying assignment for C? (The weight of the assignment is half of the number of input lines.)

Parameter: k

W[t]-hard (reduction from WEIGHTED ANTIMONOTONE q-CNF SATISFIABILITY with $q = 2$ ($t = 1$), from WEIGHTED MONOTONE t-NORMALIZED SATISFIABILITY ($t > 1$ even), and from WEIGHTED ANTIMONOTONE t-NORMALIZED SATISFIABILITY (t odd) [49])

Note: A boolean expression is *t-normalized* if it is of the form product-of-sums-of-products ... of literals with t alternations. The problem restricted to circuits having just k input lines is in FPT by the trivial algorithm that tries all possible half-weight assignments to the input lines. Actually, the problem restricted to circuits having $ks(n)$ input lines is in FPT, provided that $s(n) = o(\log n)$ [49].

HITTING SET

Instance: A finite family of sets $E = S_1, \ldots, S_n$ comprised of elements from $V = \{u_1, \ldots, u_m\}$ (equivalently, an hypergraph G with vertices in V and hyperedges $S_1, \ldots, S_n$); a positive integer k.

Question: Is there a subset $T \subseteq V$ of size at most k such that for all $S_i \in E$, $S_i \cap T \neq \emptyset$?

Parameter: k

W[2]-complete (hardness: reduction from SET COVER [210]; membership: direct proof)

Parameter: $|E|$

FPT (by a $O^*(2^{|E|})$ "dynamic programming on subsets" algorithm that finds a minimum hitting set [125])

Parameter: k, $\max\{|S_i \cap S_j| : 1 \leq i < j \leq n\}$

FPT (reported in [196])

Note: See also HITTING SET FOR SIZE d SETS. The general problem is equivalent to RED-BLUE DOMINATING SET [118].

HITTING SET FOR SIZE d SETS

Instance: A collection C of subsets of a set S, where each subset has size bounded by d ($d \geq 3$); a positive integer k.

Question: Does S contain a *hitting set* for C of size at most k? (An hitting set is a subset $S' \subseteq S$ such that S' contains at least one element from each set in C.)

Parameter: k

FPT (tractability has been shown initially for $d = 3$ by means of a reduction to problem kernel (D. Bryant) [81, Exercise 3.2.10]; tractability for arbitrary d has been proved in [173]; current best algorithm for arbitrary d has been devised by H. Fernau [117, 118] and has time complexity in $O\Big(\Big(\frac{d-1}{2}\Big(1+\sqrt{1+\frac{4}{(d-1)^2}}\Big)\Big)^k + n\Big)$; for $d = 3$, the current best algorithm has time complexiy in $O\big(2.179^k + n\big)$ [175, 117])

Note: See also Hitting Set. If $d = 2$, the problem reduces to Vertex Cover.

Immersion Order Test

Instance: Graphs G and H.

Question: Is $H \leq G$ in the immersion ordering?

Parameter: H

Open (reported in [81])

Independent Dominating Set

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a set $V' \subseteq V$ of cardinality k that is both an independent set and a dominating set in G?

Parameter: k

W[2]-complete ([75])

Note: This problem is equivalent to Minimum Maximal Independent Set [118]. Its parameterized dual is Maximum Minimal Vertex Cover, which is in FPT. See also Planar Independedt Dominating Set.

Independent Set

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a set $V' \subseteq V$ of cardinality k such that $\forall\, u, v \in V'$, $(u, v) \notin E$?

Parameter: k

W[1]-complete (membership is trivial; direct proof for hardness [78, 81])

Note: The problem is equivalent to CLIQUE. See also PLANAR INDEPENDENT SET.

INDUCED 3CNF SATISFIABILITY

Instance: A boolean formula φ in conjunctive normal form such that each clause has exactly three literals; a positive integer k.

Question: Is there a set of k variables and a truth table assignment to those variables that causes φ to unravel?

Parameter: k

W[P]-complete (hardness: reduction from CHAIN REACTION CLOSURE [2, 3])

INDUCED FORMULA SATISFIABILITY

Instance: A boolean formula φ; a positive integer k.

Question: Is there a set of k variables and a truth table assignment to those variables that causes φ to unravel?

Parameter: k

W[P]-complete (hardness: reduction from INDUCED 3CNF SATISFIABILITY [2])

INDUCED MINOR ORDER TESTING FOR PLANAR GRAPHS

Instance: Planar graphs G and H.

Question: Is $G \geq H$ in the induced minor order?

Parameter: H

FPT ([110])

INTEGER WEIGHTED VERTEX COVER

Instance: A graph $G = (V, E)$; a weight function $\omega : V \to \mathbb{N}^+$; a positive integer k.

Question: Is there a subset of vertices $V' \subseteq V$ of weight at most k such that every edge of G has at least one endpoint in V'?

Parameter: k

FPT (solvable in time $O(1.271^k + kn)$ [174, 61])

Note: See also Vertex Cover and Real Weighted Vertex Cover.

InvMax

Instance: A circuit C; an initial configuration $conf_0$ describing the placing of inverters on connections between gates in C; a positive integer k.

Question: Is there a subset A of the gates in C to which DeMorgan's rules can be applied such that the resulting circuits will have at least k gates without any inverters attached to their output lines?

Parameter: k

W[1]-hard (reduction from Independent Set [210])

i/o-Deterministic FST Composition

Instance: An alphabet Σ; an ordered set of k *i/o*-deterministic finite state transducers $A_1, \ldots, A_k$ on the common input and output alphabet Σ; a string $u \in \Sigma^+$ (let also define $q = \max\{|Q_i| : 1 \le i \le k\}$, where Q_i $(1 \le i \le k)$ is the state set of the FST A_i).

Question: Is there a sequence of strings $s_0, s_1, \ldots, s_k$ with $s_0 = u$ and $s_i \in \Sigma^{|u|}$ for $1 \le i \le k$ such that s_{i-1}/s_i is accepted by A_i for $1 \le i \le k$?

Parameter: k

W[t]-hard for all t (reduction from Bounded DFA Intersection [212]; it remains W[t]-hard for all t if $|\Sigma|$ is equal to 3 [212])

Parameter: q, $|u|$

W[2]-hard (reduction from Bounded DFA Intersection [212])

Parameter: k, $|u|$

W[1]-hard (reduction from Bounded DFA Intersection [212])

Parameter: k, q

FPT (reported in [212])

Parameter: $|u|$, $|\Sigma|$

FPT (from a variant of an algorithm in [211, Theorem 4.3.3, Part (3)] that uses an $|\Sigma|^{|u|}$-length bit-vector to store the intermediate sets of strings produced during the FST composition)

Parameter: q, $|\Sigma|$

Open (reported in [212])

Note: A (singleton, ϵ-free) FST is a 6-tuple $(Q, \Sigma_I, \Sigma_O, \delta, \sigma, F)$ where Q is a set of states, Σ_I and Σ_O are the input and output alphabets, $\delta : Q \times \Sigma_I \times \Sigma_O \times Q$ is a transition relation, $\sigma \in Q$ is the start state, and $F \subseteq Q$ is a set of final states. A FST is *i/o*-deterministic if for each $q \in Q$, $x \in \Sigma_I$ and $y \in \Sigma_O$, there is at most one $q' \in Q$ such that $(q, x, y, q') \in \delta$. The general problem is NP-hard, and the problem parameterized by $|\Sigma|$ alone is not in XP unless P = NP.

i/o-Deterministic FST Intersection

Instance: An input alphabet Σ_I and an output alphabet Σ_O; a set of k *i/o*-deterministic finite state transducers $A_1, \ldots, A_k$ on the common alphabets Σ_I and Σ_O; a string $u \in \Sigma_I^+$.

Question: Is there a string $s \in \Sigma_O^{|u|}$ such that the string-pair u/s is accepted by each FST A_i, $1 \le i \le k$?

Parameter: k

W[t]-hard for all t (reduction from Bounded DFA Intersection [212]; it remains W[t]-hard for all t even when $|\Sigma_I| = 1$ and $|\Sigma_O| = 2$ [212])

Parameter: q, $|u|$

W[2]-hard (reduction from Bounded DFA Intersection [212]; it remains W[2]-hard even when $|\Sigma_I| = 1$ [212])

Parameter: k, $|u|$

W[1]-hard (reduction from Bounded DFA Intersection [212]; it remains W[1]-hard even when $|\Sigma_I| = 1$ [212])

Parameter: $|u|$, $|\Sigma_O|$

FPT (reported in [212])

Parameter: k, q

FPT (reported in [212])

Parameter: q, $|\Sigma_I|$, $|\Sigma_O|$

FPT (reported in [212])

Note: See i/o-Deterministic FST Composition for a definition of *i/o-deterministic* FST. The unparameterized version of the problem is NP-hard, even either when $|\Sigma_I| = 1$ and $|\Sigma_O| = 2$ [212], or when $q = 4$ and $|\Sigma_O| = 3$ [18, Section 5.5.3].

Irredundant Set

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a set $V' \subseteq V$ of cardinality k having the property that each vertex $u \in V'$ has a private neighbor? A *private neighbor* of a vertex $u \in V'$ is a vertex $u' \in N[u]$ (possibly $u' = u$) with the property that for every vertex $v \in V' \setminus \{u\}$, $u' \notin N[v]$.

Parameter: k

W[1]-complete (membership: [77]; hardness: [84])

Note: See also Maximal Irredundant Set.

Jump Number

Instance: A poset $(P, \leq)$; a positive integer k.

Question: Is the jump number of P lesser than or equal to k?

Parameter: k

Open (reported in [81])

Large Stable Model

Instance: A finite propositional logic program P; a positive integer k.

Question: Does there exist a stable model for P of size at least $|P| - k$, where $|P|$ denotes the number of rules in P?

Parameter: k

FPT ($O\Big(2^{k+k^2} \cdot |P|\Big)$ algorithm [208])

Note: A model M of a logic program P is a *stable model* if M coincides with the minimal Herbrand model M for the the logic program P_M obtained from P by deleting (i) each rule having a negative literal $\neg v$ in its body with $v \in M$ and (ii) all negative literals in the bodies of the remaining rules [131]. See also SMALL STABLE MODEL.

t-LAYER PLANARIZATION

Instance: A graph $G = (V, E)$; positive integers k and r.

Question: Is there a partition of V in t sets $V_1, \ldots V_t$ ($t \geq 3$) and a subset of edges $C \subseteq E$ with $|C| \leq k$ such that if the vertices are placed in t parallel lines $L_1, \ldots L_t$ in the plane (where the vertices from V_i are placed on L_i), the number of edge crossings when the edges in $E \setminus C$ are drawn as straight-line segments is at most r?

Parameter: k, r

FPT (pathwidth-based algorithm in [91])

Note: This problem is a generalization of ONE-LAYER PLANARIZATION, TWO-LAYER PLANARIZATION, and ONE-SIDED CROSSING MINIMIZATION.

LINEAR ARRANGEMENT

Instance: A connected graph $G = (V, E)$; a positive integer k.

Question: Is there a one-to-one mapping $\sigma : V \rightarrow \{1, \ldots, |V|\}$ such that $\sum_{(u,v)\in E} |\sigma(u) - \sigma(v)| \leq k$?

Parameter: k

FPT (by reduction to problem kernel, which yields a $O(((1+\epsilon)k)! + p_\epsilon(|G|))$ algorithm, where p_ϵ is a polynomial whose degree depends on the choice of $\epsilon \leq 1$ [118])

Parameter: $k - |E|$

FPT ($O\big(|E| + |V| + 5.88^k\big)$ algorithm [140]; this is one of the few examples of results for problems "parameterized above guaranteed value" [118])

Parameter: $\mathrm{qd}(\overline{G})$

FPT (proved in [141]; $\mathrm{qd}(\overline{G})$ represents the minimum number of edges that have to be deleted from the complement of G in order to obtain a collection of disjoint cliques)

Note: The classical version of this problem is NP-complete [129, 118]. See also Bandwidth, Edge Average Min Linear Arrangement, Vertex Average Min Linear Arrangement, Linear Arrangement By Deleting Edges, Directed Linear Arrangement, and Linear Arrangement Generalized To A Vector p-Norm.

Linear Arrangement By Deleting Edges

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there an edge set $E' \subseteq E$ with $|E'| \leq k$ and a one-to-one mapping $\sigma : V \to \{1, \ldots, |V|\}$ such that $\sum_{(u,v) \in E \setminus E'} |\sigma(u) - \sigma(v)| = |E \setminus E'|$?

Parameter: k

FPT ($O\big(3^k\,|G|\big)$ algorithm [118])

Note: The classical version of this problem is NP-complete [118]. See also Linear Arrangement.

Linear Arrangement Generalized To A Vector p-Norm

Instance: A connected graph $G = (V, E)$, where $E = \{e_1, \ldots, e_m\}$; a positive integer k.

Question: Is there a one-to-one mapping $\sigma : V \to \{1, \ldots, |V|\}$ such that $\|\sigma_E\|_p \leq k$? The p-norm $\|\cdot\|_p$ $(1 < p < \infty)$ of a vector $(x_1, \ldots, x_n)$ is given by $\sqrt[p]{\sum_{i=1}^{n} |x_i|^p}$. For every edge $(u, v) \in E$, $\sigma_E((u, v)) = |\sigma(u) - \sigma(v)|$. Finally, $\|\sigma_E\|_p$ is the p-norm of the vector $(\sigma_E(e_1), \ldots, \sigma_E(e_m))$.

Parameter: k

FPT (by reduction to a problem kernel of size k^p [118]. The same result holds when the edges are associated with integer weights ≥ 1, and when replacing edges with hyperedges [118].)

Note: The complexity of the classical version of this problem for $p \notin \{1, \infty\}$ is open [150]. For $p = 1$ this problem reduces to Linear Arrangement, and for $p = \infty$ (maximum function) it reduces to Bandwidth. The parameterized complexity of the version in which p $(\neq \infty)$ is part of the instance is open [118].

Linear Extension Count

Instance: A poset $(P, \leq)$; a positive integer k.

Question: Does P have at least k linear extensions?

Parameter: k

randomized FPT ([39, 40])

LINEAR INEQUALITIES

Instance: A system of linear inequalities; a positive integer k.

Question: Can we delete k of the inequalities and get a system that is consistent over the rationals?

Parameter: k

W[P]-complete (hardness: reduction from WEIGHTED MONOTONE CIRCUIT SATISFIABILITY [2, 3])

LOG CIRCUIT SATISFIABILITY

Instance: A boolean circuit C.

Question: Does C have a satisfying assignment?

Parameter: $\lceil |X| \,/\, \log |C| \rceil$

M[P]-complete ([103, 104, 73, 123])

LOG FORMULA SATISFIABILITY

Instance: A boolean expression F with variables in X.

Question: Does F have a satisfying truth assignment?

Parameter: $\lceil |X| \,/\, \log |F| \rceil$

M[SAT]-complete ([103, 104, 73, 123])

LOG VERTEX COVER

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a vertex cover $C \subseteq V$ with $|C| \leq k \log(|V|)$?

Parameter: k

M[1]-complete ([103, 104, 73, 123])

Note: See also VERTEX COVER and MINIATURIZED VERTEX COVER.

Long Cycle

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does G have a cycle of length at least k?

Parameter: k

FPT (Fellows and Langston [114])

Note: See also Exact Long Cycle, which requires a different proof technique.

Longest Common Subsequence

Instance: An alphabet Σ; a set of k strings $r_1, \ldots, r_k$ over the alphabet Σ; a positive integer λ.

Question: Is there a string $s \in \Sigma^*$ of length at least λ that is a subsequence of each r_i, for $i = 1, \ldots, k$? (A string s is a *subsequence* of a string r if we can delete some characters in r such that the remaining string is equal to s.)

Parameter: k

W[t]-hard for all t (reduction from Weighted Monotone t-Normalized Satisfiability [81, 29, 28])

Parameter: k, $|\Sigma|$

W[t]-hard for all t (reduction from Longest Common Subsequence parameterized by k [27])

Parameter: λ

W[2]-hard, in W[P] (membership is easy; hardness: reduction from Dominating Set [28], [81, Exercise 12.0.6]; in FPT if $|\Sigma|$ is parameter, by the trivial algorithm that generates all $|\Sigma|^\lambda$ possible subsequence strings and checks them against each r_i)

Parameter: k, λ

W[1]-complete (membership: reduction to Weighted q-CNF Satisfiability [81, 29, 28]; hardness: reduction from Clique [81, 29, 28])

Parameter: k, max. number of occurrences of each character in each string

FPT ($O\big(2^{2k \log h} \cdot k \cdot n^2\big)$ algorithm in [137], where h denotes the maximum number of occurrences of each character $a \in \Sigma$ in each string $r_1, \ldots, r_k$)

Note: See also Fixed Alphabet Longest Common Subsequence.

Matrix Domination

Instance: An $n \times n$ boolean matrix M; a positive integer k.

Question: Is there a set C of k or fewer nonzero entries in M that dominate all others, in the sense that every nonzero entry in M is in the same row or in the same column as some element of C?

Parameter: k

FPT (by a combination of search tree and reduction to problem kernel methods [81, Exercise 3.2.9]; also, in FPT by means of a fixed-parameter reduction to Edge Dominating Set [214], which yields a $O^*(2.6181^k)$ algorithm [118])

Matrix Row Column Merging

Instance: An matrix $M \in \{0,1\}^{n \times m}$; a positive integer k.

Question: Is it possible to get the all-zeros matrix by merging at most k neighboring rows or columns of M? The *merging* operation performs a component-wise logical AND.

Parameter: k

FPT (by a $O(2.6181^k\, mn)$ algorithm in [116])

Note: NP-completeness of this problem is proved in [116].

Maximal Irredundant Set

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a set $V' \subseteq V$ of cardinality k such that (1) each vertex $u \in V'$ has a private neighbor and (2) V' is not a proper subset of any $V'' \subseteq V$ that also has this property? A *private neighbor* of a vertex $u \in V'$ is a vertex $u' \in N[u]$ (possibly $u' = u$) with the property that for every vertex $v \in V' - \{u\}$, $u' \notin N[v]$.

Parameter: k

W[2]-hard, in W[P] (membership: reduction to Bounded Non-determinism Turing Machine Computation [53]; hardness: reduction from Dominating Set [26])

Note: See also Irredundant Set.

Maximum Agreement Subtree

Instance: A set $\{T_1, \ldots, T_n\}$ of binary rooted trees with equal label set L; a positive integer k.

Question: Is there a set of labels $\Lambda \subseteq L$, $|\Lambda| \leq k$, such that all trees $T_i \setminus \Lambda$ are isomorphic?

Parameter: k

FPT (by a reduction to Hitting Set For Size d Sets with $d = 3$ [118])

Note: A *phylogeny* is a rooted tree T whose leaf set is in bijection with a label set L. In this problem the rooted tree is always binary, i.e., all inner nodes but the root have degree three; the root has degree two. To maintain this property, label deletion propagates as follows: Assume that x is an inner node with children y and z, where z is a leaf that is going to be deleted. Upon deleting z, x would have a degree that is too small. Therefore, x is deleted as well. Two subcases arise: (1) x is the root of the tree: then, y will be the root of the tree that is produced. (2) x is not the root of the tree: then, x is the child of another node x', and an edge between x' and y is added in the tree that is produced to maintain it connected. The notation $T \setminus \Lambda$ denotes the tree that is produced from T by deleting all labels in the set Λ.

Maximum Cut

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a cut set $C \subseteq E$ with $|C| \geq k$? A *cut set* C is a subset of edges such that $(V, E \setminus C)$ is a bipartite graph.

Parameter: k

FPT (reduction to problem kernel of $2k$ edges [166]; kernels with $2k$ edges have been also obtained with different methods in [163, 185]; current best algorithm has running time in $O\left(k \cdot 2^{k/2} + |V|^2\right)$ [185])

Parameter: $k - \lceil |E| / 2 \rceil$

FPT ([166, 185, 187])

Note: The parametric dual of this problem is Bipartization By Edge Removal. The current best exact algorithm for this problem has running time in $O^*\left(2^{|E|/4}\right)$ [100, 101].

MAXIMUM KNAPSACK

Instance: A set of n items $\{x_1, \ldots, x_n\}$ with sizes $s_1, \ldots, s_n$ and profits $p_1, \ldots, p_n$, the knapsack capacity b, and the profit threshold k (all numbers are encoded in binary).

Question: Is there a subset of items that yields a profit larger than k and has overall size less than b?

Parameter: k

FPT (actually, in *efficient*-FPT because the classical problem admits a fully polynomial-time approximation scheme (FPTAS) [15, 59]; also in [45])

Parameter: b

FPT (reduction to problem kernel [118])

MAXIMUM LEAF SPANNING TREE

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does G have a spanning tree with k or more leaves?

Parameter: k

FPT (Downey and Fellows [81], correcting [79]; also Bodlaender [24]; also in LOGSPACE+*advice* [47]. A fast FPT algorithm is in [37]. The current best algorithm has time complexity in $O^*(9.4815^k)$ [187].)

MAXIMUM LIKELIHOOD DECODING

Instance: A binary $m \times n$ matrix H; a target vector $s \in \mathbf{F}^m$; a positive integer k.

Question: Is there a set of at most k columns of H that sum to s?

Parameter: k

W[1]-hard, in W[2] ([89])

Note: See also WEIGHT DISTRIBUTION.

MAXIMUM MINIMAL VERTEX COVER

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does there exist a minimal vertex cover set of cardinality $\geq k$?

Parameter: k

FPT ($O^*(2^k)$ algorithm in [118]; the problem restricted to planar graphs admits a problem kernel of size $4k-4$ [118])

Note: The paramerized dual of this problem is INDEPENDENT DOMINATING SET, which is W[2]-complete.

MAXIMUM SATISFIABILITY

Instance: A Boolean formula F in conjunctive normal form (CNF), with m clauses and variables in X; a positive integer k.

Question: Is there an assignment $\alpha : X \to \{\text{TRUE}, \text{FALSE}\}$ such that at least k clauses in F evaluate to TRUE under α?

Parameter: k

FPT (by a trivial reduction to problem kernel of size $2k$ [166])

Parameter: $k - m/2$

FPT (by a search tree algorithm in [166], then improved in [134])

MINIATURIZED CIRCUIT SATISFIABILITY

Instance: A Boolean circuit C; a positive integer m encoded in unary.

Question: Is there a setting of the inputs that cause C to output 1?

Parameter: $\lceil |C| / \log m \rceil$

M[1]-complete ([123])

MINIATURIZED d-CNF SATISFIABILITY

Instance: A Boolean formula F in conjunctive normal form (CNF) with variables in X and each clause of at most d literals ($d \geq 3$); a positive integer m encoded in unary.

Question: Is there a satisfying assignment $\alpha : X \to \{\text{TRUE}, \text{FALSE}\}$ for F?

Parameter: $\lceil |F| / \log m \rceil$

M[1]-complete ([49, 73, 123])

Note: See also 3-CNF SATISFIABILITY.

MINIATURIZED d-COLORABILITY

Instance: A graph $G = (V, E)$; a positive integer m encoded in unary.

Question: Is G d-colorable, that is, does there exists a function $c : V \to \{1, \ldots, d\}$ such that $\forall (v, w) \in E$, $c(v) \neq c(w)$?

Parameter: $\lceil |G| / \log m \rceil$

M[1]-complete ([123])

Note: The classical non-miniaturized version of this problem is NP-complete for any $d \geq 3$ [130].

MINIATURIZED CLIQUE

Instance: A graph $G = (V, E)$; a positive integer k; a positive integer m encoded in unary.

Question: Is there a clique $C \subseteq V$ with $|C| \leq k$?

Parameter: $\lceil |G| / \log m \rceil$

FPT (trivial corollary of a general theorem: a miniaturized parameterized problem is in FPT if and only if the classical (non-miniaturized) version of the same problem is in DTIME($2^{o^{\text{eff}}(n)}$), where n denotes the length of the instance of the problem [123])

Note: See also CLIQUE. Notice that MINIATURIZED INDEPENDENT SET is M[1]-complete.

MINIATURIZED INDEPENDENT SET

Instance: A graph $G = (V, E)$; a positive integer k; a positive integer m encoded in unary.

Question: Is there an independent set $C \subseteq V$ with $|C| \leq k$?

Parameter: $\lceil |G| / \log m \rceil$

M[1]-complete ([103, 104, 73, 123])

Note: See also INDEPENDENT SET. Notice that MINIATURIZED CLIQUE is in FPT.

MINIATURIZED VERTEX COVER

Instance: A graph $G = (V, E)$; a positive integer k; a positive integer m encoded in unary.

Question: Is there a vertex cover $C \subseteq V$ with $|C| \leq k$?

Parameter: $\lceil |G| \,/\, \log m \rceil$

M[1]-complete ([103, 104, 73, 123])

Note: See also Vertex Cover and Log Vertex Cover.

Miniaturized Weighted Circuit Satisfiability

Instance: A boolean circuit C; a positive integer k; a positive integer m encoded in unary.

Question: Does C have a satisfying assignment of Hamming weight k?

Parameter: $\lceil |C| \,/\, \log m \rceil$

M[1]-complete ([103, 104, 73, 123])

Note: See also Weighted Circuit Satisfiability.

Miniaturized Weighted q-CNF Satisfiability

Instance: A boolean expression X in conjunctive normal form (CNF) such that each clause has no more than q literals ($q \geq 2$); a positive integer k; a positive integer m encoded in unary.

Question: Does X have a satisfying truth assignment of weight k?

Parameter: $\lceil |X| \,/\, \log m \rceil$

M[1]-complete ([103, 104, 73, 123])

Note: See also Weighted q-CNF Satisfiability.

Minimal Diagnosis

Instance: A finite set of faults F; a set of effects M; a function $e : F \to 2^M$ relating faults and effects; a set of observed effects $M' \subseteq M$; a positive integer k.

Question: Is there a set $F' \subseteq F$ with $|F'| \leq k$ such that $M' \subseteq \bigcup_{f \in F'} e(f)$?

Parameter: $|e|$

FPT (by building a two-column exponential-size table containing the set of effects corresponding to every subset of failures and, accordingly, the size of the minimal diagnosis [160])

Parameter: $|M'|$

FPT ([118])

Note: As noted in [118], this problem is essentially a reparameterization of Hitting Set.

Minimum Axiom Set

Instance: A finite set S of "sentences"; an "implication relation" R consisting of pairs (A, t) where $A \subseteq S$ and $t \in S$; a positive integer k.

Question: Is there a set $S_0 \subseteq S$ with $|S_0| \leq k$ and a positive integer n such that if we define S_i, $1 \leq i \leq n$, to consist of exactly those $t \in S$ for which either $t \in S_{i-1}$ or there exists a set $U \subseteq S_{i-i}$ such that $(U, t) \in R$, then $S_n = S$?

Parameter: k

W[P]-complete (membership: reduction to Weighted Monotone Circuit Satisfiability [2]; hardness: reduction from Weighted Circuit Satisfiability [2, 83, 82])

Minimum Degree Graph Partition

Instance: A graph $G = (V, E)$; positive integers k and d.

Question: Can V be partitioned into disjoint subsets $V_1, \ldots, V_m$ so that, for $1 \leq i \leq m$, $|V_i| \leq k$ and at most d edges have exactly one endpoint in V_i?

Parameter: k, d

Open (reported in [81])

Minimum Disjunctive Normal Form

Instance: A set $X = \{x_1, x_2, \ldots, x_n\}$ of variables; a set $A \subseteq \{0, 1\}^n$ of implicants; a positive integer m.

Question: Is there a Disjunctive Normal Form expression E over X, having no more than m disjuncts, such that E is true for precisely those truth assignments in A and no others?

Parameter: $|A|$

FPT (solvable in $O\big(2^{|A|}n\big)$ time [83, 82])

Minimum Fill-in

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Can we add no more than k edges to G and cause G to become chordal?

Parameter: k

FPT (solvable in time $O(c^k \cdot |E|)$ and $O(k^5 \cdot |E| \cdot |V| + f(k))$ [155])

Minimum Inner Node Spanning Tree

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a spanning tree of G with at most k inner nodes?

Parameter: k

W[2]-hard ([103]. Equivalent to Connected Dominating Set, and dual of Maximum Leaf Spanning Tree [118].)

Minimum Maximal Independent Set

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does there exist a maximal independent set of cardinality $\leq k$? A maximal independent set is a subset $U \subseteq V$ such that if $u, v \in U$ then $(u, v) \notin E$ and $\forall\, w \in V - U$, there exists $u \in U$ such that $(u, w) \in E$.

Parameter: k

W[2]-complete (equivalent to Independent Dominating Set [118])

Minimum Partition

Instance: A finite set $X = \{x_1, \ldots, x_n\}$; a weight function $w : X \to \mathbb{R}_{\geq 1}$; an integer k.

Question: Is there a set $Y \subset X$ such that $\max\left\{\sum_{y \in Y} w(y), \sum_{z \notin Y} w(z)\right\} \leq k$?

Parameter: k

FPT (reduction to problem kernel [118])

Minimum Quartet Inconsistency

Instance: A set S of n taxa; a set Q_S of $\binom{n}{4}$ quartet topologies such that there is exactly one topology for every quartet corresponding to S; a positive integer k.

Question: Is there an evolutionary tree T where the leaves are bijectively labeled by the elements from S such that the set of quartet topologies induced by T differs from Q_S in at most k quartet topologies?

Parameter: k

FPT (solvable in time $O(4^k n + n^4)$ [135]; observe that the input size is $O(n^4)$)

Note: An *evolutionary tree* for S is a binary tree T in which the leaves are bijectively labeled by the set of taxa S; a *quartet* is a size four subset $\{a, b, c, d\} \subseteq S$. The topology for $\{a, b, c, d\}$ induced by T is the four-leaves subtree of T induced by $\{a, b, c, d\}$; for every quartet there are three possible topologies. The general problem is NP-complete [19, 152], but it is solvable in polynomial time if $k < (n-3)/2$ [200].

Minimum Terminal Pair Separation

Instance: A graph $G = (V, E)$; pairs of vertices (s_1, t_1), (s_2, t_2), ..., (s_ℓ, t_ℓ); an integer k.

Question: Is there a set of vertices $S \subseteq V$ of size at most k such that for every $1 \leq i \leq \ell$, vertices s_i and t_i are in different connected components of $G \setminus S$?

Parameter: k

Open (reported in [167])

Parameter: k, ℓ

FPT ($O^*\left(2^{k\ell} k^{k+1} 4^{k^3}\right)$ search tree algorithm in [167])

Note: This problem and its edge-deletion variant are equivalent. The classical problem is NP-hard even for $\ell = 3$ [64]. See also Minimum Terminal Separation.

Minimum Terminal Separation

Instance: A graph $G = (V, E)$; a set of terminals $T \subseteq V$; an integer k.

Question: Is there a set of vertices $S \subseteq V$ of size at most k such that no two vertices of T belongs to the same connected component of $G \setminus S$?

Parameter: k

FPT ($O^*\left(k^{k+1}4^{k^3}\right)$ search tree algorithm in [167])

Note: This problem and its edge-deletion variant are equivalent. The classical problem is NP-hard even for $|T| = 3$ [64]. See also MINIMUM TERMINAL PAIR SEPARATION.

MINOR ORDER TEST

Instance: Graphs $G = (V, E)$ and $H = (U, A)$.

Question: Is H a *minor* of G?

Parameter: H

FPT (solvable in $O\left(|V|^3\right)$ for fixed H [193])

MODULE ALLOCATION ON GRAPHS OF BOUNDED TREEWIDTH

Instance: A set of modules $M = \{1, 2, \ldots, m\}$; a set of processors $P = \{1, 2, \ldots, p\}$; a cost function $e : (M \times P) \to \mathbb{R}$, where $e(x, y)$ is the cost of executing module $x \in M$ on processor $y \in P$; a communication cost function $C : (M \times P \times M \times P) \to \mathbb{R}$ where $C(x, y, x', y')$ is the communication cost when module x is assigned to processor y and module x' is assigned to processor y'; a communication graph $G = (M, E)$; a positive real number l.

Question: Does there exist an assignment of modules to processors such that the total cost of execution is less than or equal to l?

Parameter: treewidth(G)

W[t]-hard for all t ([31, 33])

MONOCHROME CYCLE COVER

Instance: An edge-colored graph $G = (V, E)$; a positive integer k.

Question: Is there a set of k vertices $V' \subseteq V$ with the property that every monochrome cycle in G contains a vertex in V'?

Parameter: k

W[2]-hard, in W[P] ([81, Appendix A] reports that this problem is W[2]-hard and in W[P] by Downey and Fellows (unpublished))

Monoid Factorization

Instance: A set A of self-maps on $[n]$; a self-map h on $[n]$; an integer k.

Question: Is there a factorization of h of length k over A?

Parameter: k

W[2]-hard, in W[P] (membership: direct proof ? [48]; hardness: reduction from Dominating Set [48])

More Than Half MAX-3SAT

Instance: An expression E in conjunctive normal form such that each clause has exactly 3 literals; a positive integer k.

Question: Is there a truth assignment that satisfies at least $\lceil \frac{n}{2} \rceil + k$ clauses of E?

Parameter: k

FPT (by the reduction to problem kernel method (V. Raman) [81, Exercise 3.2.8])

Multidimensional Matching

Instance: A set $M \subseteq X_1 \times \ldots \times X_r$, where the X_i are disjoint sets; a positive integer k.

Question: Is there a subset $M' \subseteq M$ with $|M'| = k$, such that no two elements of M' agree in any coordinate?

Parameter: r, k

FPT (by the perfect hashing method [81])

Multi-Hitting Set For Size d Sets

Instance: A hypergraph $G = (V, E)$, with each hyperedge of size at most d; positive integers k and ℓ.

Question: Is there a *multi-hitting set* $C \subseteq V$ with $|C| \leq k$, that is, C satisfies $\forall e \in E\ \exists c \subseteq e$ s.t. $(|c| \geq \ell \wedge c \subseteq C)$?

Parameter: k, ℓ

FPT ($O^*(d^k)$ search tree algorithm in [118])

Note: The variant where d is arbitrarily large is W[2]-hard by the trivial reduction from Hitting Set.

Nearest Vector

Instance: A basis $X = \{x_1, \ldots, x_n\} \subset \mathbb{Z}^n$ for a lattice Ł; a target vector $s \in \mathbb{Z}^n$; a positive integer k.

Question: Is there a vector $x \in$ Ł such that $\|x - s\|^2 \leq k$?

Parameter: k

W[1]-hard (reduction from Perfect Code [89])

Note: See also Theta Series.

Nearly A Partition

Instance: A finite set X; a family F of subsets of X; a positive integer k.

Question: Is there a subfamily $F' \subseteq F$ with $|F'| \leq k$ such that $F - F'$ is a partition of X?

Parameter: k

FPT (by a combination of search tree and reduction to problem kernel methods [81, Exercise 3.2.9])

Nonblocker Set

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a nonblocker set $N \subseteq V$ with $|N| \geq k$? N is a *nonblocker set* if, for each $v \in V$, there exists $u \notin N$ such that $(u, v) \in E$.

Parameter: k

FPT (by reduction to a problem kernel of at most $5/3\,k$ vertices, which yields a $O^*(3.0701^k)$ algorithm [118, unpublished result by F. Dehne, M. Fellows, E. Prieto, F. Rosamond]; this result can be improved to a $O^*(3.0121^k)$ algorithm for general graphs, and to a $O^*(2.4932^k)$ algorithm for bipartite graphs [125, 118]; finally, by using exponential space, the problem can be solved in time and space $O^*(2.5154^k)$ [118])

Note: This problem is the parameterized dual of Dominating Set when taking as size function the number of vertices of G. Essentially, the same reduction to problem kernel works for a reparameterization of this problem, in which the nonblocker set must be larger than or equal to $\delta\,|V| + k$, where $\delta \in (0, 3/5)$ is a fixed real constant [118].

Non-Uniform Register Allocation

Instance: A structured program P having dynamically allocated variables of types $t_1, \ldots, t_k$ ($k \geq 2$); positive integers $w_1, \ldots, w_k$.

Question: Can P be compiled in such a way that all dynamically allocated variables of type t_i are assigned to at most w_i hardware registers ($1 \leq i \leq k$) without spilling?

Parameter: $w_1, \ldots, w_k$ (without rescheduling)

FPT (solvable in time $O(f(k) \cdot |P|)$ [22])

Parameter: $w_1, \ldots, w_k$ (with rescheduling)

W[t]-hard for all t (reduction from 2-Colored Directed Vertex Separation Number [22])

Note: A *structured program* is a program encoded in a high-level language without *goto*s. *Spilling* is the act of temporarily saving some variables on the stack in order to free the corresponding hardware registers. Compilers may or may not reorder the program statements (*rescheduling*), as long as the data dependencies are not violated. For $k = 1$, refer to Uniform Register Allocation.

Odd Set

Instance: A red/blue graph $G = (\mathcal{R}, \mathcal{B}, E)$; a positive integer k.

Question: Is there a set of at most k vertices $R \subseteq \mathcal{R}$ such that each member of $\mathcal{B}$ has an odd number of neighbours in R?

Parameter: k

W[1]-hard (reduction from Perfect Code [89])

Note: See also Exact Odd Set, Exact Even Set, Even Set.

One-Layer Planarization

Instance: A bipartite graph $G = (V_1, V_2, E)$; a linear ordering $<$ on V_1; a positive integer k.

Question: Is there a set $C \subseteq E$, $|C| \leq k$, whose removal allows a biplanar drawing of the graph that respects $<$ on V_1?

Parameter: k

FPT ($O\left(k^3 \cdot 2.5616^k + |G|^2\right)$ algorithm in [92, 118])

Note: A bipartite graph $G = (V_1, V_2, E)$ is *biplanar* if the vertices can be placed on two parallel lines L_1 and L_2 in the plane (where the vertices from V_i are placed on L_i) such that there are no edge crossings when edges are drawn as straight-line segments. See also Two-Layer Planarization and One-Sided Crossing Minimization.

One-Sided Crossing Minimization

Instance: A bipartite graph $G = (V_1, V_2, E)$; a linear order $\prec_1$ on V_1; a positive integer k.

Question: Is there a linear order $\prec$ on V_2 such that, when the vertices from V_1 are placed on a line (also called layer) L_1 in the order induced by $\prec_1$ and the vertices from V_2 are placed on a second layer L_2 (parallel to L_1) in the order induced by $\prec$, then drawing straight lines for each edge in E will introduce no more than k edge crossings?

Parameter: k

FPT (results in [91, 92, 94, 95, 93]; current best algorithm has time complexity in $O^*\left(1.4656^k\right)$ [93])

One-Tree Drawing By Deleting Edges

Instance: A binary tree T with leaf labels Λ; a linear ordering $\prec$ on Λ; a positive integer k.

Question: Is there a label set $L \subseteq \Lambda$ with $|L| \leq k$ such that the tree produced by deleting the leaves associated with labels in L can be drawn without crossing in the plane, so that the leaves in $\Lambda \setminus L$ are arranged according to the ordering $\prec$ on some line?

Parameter: k

FPT ($O^*(2.179^k)$ algorithm in [118])

PARTITIONED CLIQUE

Instance: A graph $G = (V, E)$; an integer k; a partition $\{U_1, \ldots, U_k\}$ of V into k sets of equal size ($|V|$ must be a multiple of k).

Question: Is there a subset $V' \subseteq V$ of cardinality k such that for each $u, v \in V'$, $(u, v) \in E$ and for each $1 \leq i \leq k$, $|V' \cap U_i| = 1$?

Parameter: k

W[1]-complete (reduction from and to CLIQUE [182])

3-PATH PACKING

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Are there k vertex disjoint instances of $K_{1,2}$ (i.e., a path of length three) in G?

Parameter: k

FPT (by a reduction to problem kernel of size $15k$ yielding a $O^*(2^{5.3k})$ algorithm [186])

Note: See also TRIANGLE PACKING, s-STAR PACKING, and GRAPH PACKING.

PATHWIDTH

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is the pathwidth of G no more than k?

Parameter: k

FPT (Fellows and Langston [113]; also Bodlaender's $O(f(k) \cdot n)$ algorithm [23])

Note: Equivalent to GATE MATRIX LAYOUT.

PEBBLE GAME

Instance: A pebble game (N, R, S, T); a positive integer k.

Question: If $|S| = k$ determine if I has a winning strategy

Parameter: k

XP-complete (membership: direct proof [81]; hardness: direct proof [5, 6, 81])

Note: A Pebble Game is a quadruple (N, R, S, T) consisting of a finite set of vertices N, a set of rules $R \subseteq \{(x, y, z) \mid x, y, z \in N, x \neq y, y \neq z\}$, the start set $S \subset N$, the terminal vertex $T \in N$. The game is played by Player I and Player II, who alternatively move pebbles on the nodes. At the beginning of the game, pebbles are placed on all the start nodes. If $(x, y, z) \in R$, and there are pebbles upon x and y but not on z, then a Player can move a pebble from x to z. The winner is the first Player to put a pebble on T or can force the opponent into a position where the opponent cannot move.

Peg Game

Instance: A peg game $G = (V, k, l)$; a positive integer k.

Question: Does Player I have a winning strategy?

Parameter: k

XP-complete (membership: direct proof [81]; hardness: reduction from Pebble Game [5, 6])

Note: Let Q be a set of integers. The Peg Game is a triple $G = (V, k, l)$ with $k, l \in Q$ and $V \subseteq V^l$ such that $(v_1, \ldots, v_l) \in V^l$ implies that $v_1 + \ldots + v_l = 0$. A play of the Peg Game is as follows. There are l pegs on the board and k rings. The interpretation of the vector $(v_1, \ldots, v_l) \in V$ is that for each i, if $v_i \geq 0$, then we put v_i rings on the ith peg, and if $v_i \leq 0$, we remove v_i rings from the ith peg. Initially, all k rings are on the first peg. Players play alternatively according to the rules V of the game. The Player who wins is the one who places all k rings on the last peg, or who forces the opponent into a position where the opponent has no valid play.

Perfect Code

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does G have a k-element perfect code? A *perfect code* is a set of vertices $V' \subseteq V$ with the property that for each vertex $v \in V$, there is precisely one vertex in $N[v] \cap V'$.

Parameter: k

W[1]-complete (membership: reduction to Short Nondeterministic Turing Machine Computation [52, 53]; hardness: reduction from Independent Set [78])

k-Perfect Matching

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does G have at least (or exactly) k perfect matchings?

Parameter: k

FPT (solvable in time $O(k \cdot |E|)$ [151])

Note: For *weighted* graphs, finding the best k matchings is FPT by, for instance, Chegireddy and Hamacher [58].

Perfect Phylogeny

Instance: A set $\mathcal{C} = \{1, \ldots, m\}$ of characters; for each $c \in \mathcal{C}$, a set $A_c = \langle 1, \ldots, r_c \rangle$ of states; a set $\mathcal{S}$ of n species $s \in A_1 \times \ldots \times A_m$.

Question: Is there a tree $\mathcal{T}$ with the properties: (i) $\mathcal{S} \subseteq V(\mathcal{T}) \subseteq A_1 \times \ldots \times A_m$ (ii) every leaf in $\mathcal{T}$ is in $\mathcal{S}$ (iii) for each $c \in \mathcal{C}$ and each $j \in A_c$, the set of vectors $v \in V(\mathcal{T})$ such that $v_c = j$ induces a subtree of $\mathcal{T}$?

Parameter: $r = \max_{c \in \mathcal{C}} r_c$

FPT ($O\big(2^{3r}(nm^3 + m^4)\big)$ algorithm [7])

Parameter: $r = \max_{c \in \mathcal{C}} r_c$, m

FPT ($O((r - n/m)^m rnm)$ algorithm [8])

Note: In the Compendium in [81], the meaning of r is not specified, and $\max_{c \in \mathcal{C}} r_c$ is defined to be m. To be checked. This problem is also known as Triangulating Colored Graphs.

Permanent Lower Bound

Instance: A binary matrix M; a positive integer k.

Question: Does the permanent of M exceed k?

Parameter: k

FPT (equivalent to Bipartite Matching Cardinality [151])

Permutation Group Factorization

Instance: A set $A \subseteq S_n$ of permutations of n objects; a permutation $x \in S_n$; a positive integer k.

Question: Does x have a factorization of length k over A?

Parameter: k

W[1]-hard, in W[P] (membership: direct proof [48]; hardness: reduction from PERFECT CODE [48])

PLANAR ANNOTATED DOMINATING SET

Instance: A planar *black* and *white* graph $G = (B \cup W, E)$; a positive integer k.

Question: Is there a set of k vertices $V' \subseteq B \cup W$ with the property that for every vertex $v \in B$ there is a vertex $v' \in N[v] \cap V'$?

Parameter: k

FPT (Alber, Fan, Fellows, Fernau, Niedermeier, Rosamond, and Stege $O(8^k \cdot |G|)$ algorithm [11])

Note: See also PLANAR DOMINATING SET.

PLANAR CLIQUE

Instance: A planar graph $G = (V, E)$; a positive integer k.

Question: Is there a set $V' \subseteq V$ of cardinality k such that $\forall u, v \in V'$, $(u, v) \in E$?

Parameter: k

FPT (trivial for $k \geq 5$ due to Kuratowski's Theorem [71])

Note: See also CLIQUE.

PLANAR CONNECTED DOMINATING SET

Instance: A planar graph $G = (V, E)$; a positive integer k.

Question: Is there a subset $D \subseteq V$ with $|D| \leq k$ such that D is both a connected set and a dominating set?

Parameter: k

FPT ([118])

PLANAR DIGRAPH KERNEL

Instance: A planar digraph $D = (V, A)$; a positive integer k.

Question: Does there exist a kernel in D of size at most k? A *kernel* is a set of nodes S such that S is independent and for every vertex $x \in V \setminus S$, there is $y \in S$ such that $xy \in A$.

Parameter: k

FPT (by a $O\left(|V|\,2^{19.1\sqrt{k}} + |V|^4\right)$ algorithm in [139]; [139] includes also a $O\left(|V|^2 + 2^{19.1\sqrt{k}} k^9\right)$ algorithm)

Note: See also Digraph Kernel.

PLANAR DIRECTED DISJOINT PATHS

Instance: A directed planar graph $G = (V, A)$; k pairs $\langle r_1, s_1\rangle, \ldots, \langle r_k, s_k\rangle$.

Question: Does G have k vertex-disjoint paths $P_1, \ldots, P_k$ with P_i running from r_i to s_i?

Parameter: k

Open (reported in [81])

PLANAR DOMINATING SET

Instance: A planar graph $G = (V, E)$; a positive integer k.

Question: Is there a set of k vertices $V' \subseteq V$ with the property that every vertex of G either belongs to V' or has neighbor in V'?

Parameter: k

FPT (from a $O\left(8^k\, k + n^3\right)$ algorithm [11], where $n = |V|$; improved to $O\left(2^{O(\sqrt{k})} n\right)$ algorithm [9, 12, 13, 11]; the current best algorithms have running time $O\left(2^{16.4715\sqrt{k}} + n^3\right)$ [153, 118] and $O\left(2^{15.13\sqrt{k}} n\right)$ [126]; it does not have a $O\left(2^{o(\sqrt{k})}\mathrm{poly}(n)\right)$ algorithm unless all MAXSNP-complete problems can be solved in time $O\left(2^{o(k)}\mathrm{poly}(n)\right)$ [49])

Note: See also Dominating Set. Previously claimed to be in FPT by a flawed $O\left(11^k n\right)$ algorithm [81]. Also [10] describes a $O\left(2^{O(\sqrt{k})} n\right)$ algorithm, but the constants of the time complexity are wrong.

Planar Domination Improvement Number

Instance: A planar graph $G = (V, E)$; a positive integer k.

Question: Is there a planar graph G' with $G \subseteq G'$, such that G' has a k-element dominating set?

Parameter: k

FPT (Dejter and Fellows' application of the Robertson-Seymour Theorem [70])

Planar Embedding Face Cover

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Can G be embedded in the plane so that there are k faces that cover all vertices?

Parameter: k

FPT (solvable in $O(f(k) \cdot n)$ time [20])

Note: Is this problem equivalent to Face Cover?

Planar Feedback Vertex Set

Instance: A planar graph $G = (V, E)$; a positive integer k.

Question: Is there a set U of at most k vertices of G such that each cycle of G passes through some vertex of U?

Parameter: k

FPT ($O^*\big(4.5414^k + n^2\big)$ algorithm [118])

Note: See also Feedback Vertex Set.

Planar Independendet Dominating Set

Instance: A planar graph $G = (V, E)$; a positive integer k.

Question: Is there an independent dominating set $D \subseteq V$ with $|D| \leq k$?

Parameter: k

FPT (original proof in [79]; $O^*\big(5.1623^k\big)$ algorithm in [118])

Note: See also Independent Dominating Set.

Planar Independent Set

Instance: A planar graph $G = (V, E)$; a positive integer k.

Question: Is there a set $V' \subseteq V$ of cardinality k such that $\forall u, v \in V'$, $(u, v) \notin E$?

Parameter: k

FPT (Lipton and Tarjan approximation algorithm [161, 162]; it does not have a $O\left(2^{o(\sqrt{k})}\text{poly}(n)\right)$ algorithm ($n = |V|$) unless 3-SAT $\in$ DTIME$(2^{o(n)})$ [49]; also solvable by means of "greedy kernelization" [118], or by computing a four-color map of the graph, which yields a problem kernel of size $4k - 4$ [118] that, combined with a general algorithm having time complexity $O^*\left(2^{|V|/4}\right)$ [194, 195], yields a parameterized algorithm with running time in $O^*\left(2^k\right)$; the best current algorithm is treewidth-based and has running time in $O\left(2^{4\sqrt{3k}}n\right)$ [118])

Note: See also Independent Set.

Planar Maximum Minimal Dominating Set

Instance: A planar graph $G = (V, E)$; a positive integer k.

Question: Does there exist a minimal dominating set of cardinality $\geq k$?

Parameter: k

FPT (by a greedy reduction to a problem kernel of size $4k - 4$ [118])

Planar Multiway Cut

Instance: An edge-weighted planar graph $G = (V, E)$; a set of terminals $\{x_1, \ldots, x_k\} \subseteq V$; a positive integer M.

Question: Is there a set of edges of total weight at most M whose removal disconnects each terminal from all the others?

Parameter: k

Open (reported in [65, 81])

Planar Red-Blue Dominating Set

Instance: A planar graph $G = (V, E)$ with V partitioned in $V_{\text{red}} \cup V_{\text{blue}}$; a positive integer k.

Question: Is there a subset $D \subseteq V_{\text{red}}$ with $|D| \leq k$ and $V_{\text{blue}} \subseteq N(D)$ (that is, $\forall\, v \in V_{\text{blue}}, \exists\, w \in D$ such that $(v, w) \in E$)?

Parameter: k

FPT (by a reduction to Annotated Face Cover [118]; current best algorithm has running time in $O\left(2^{24.551\sqrt{k}}\, n\right)$ [118])

Note: See also Red-Blue Dominating Set.

Planar Roman Domination

Instance: A planar graph $G = (V, E)$; a positive integer k.

Question: Is there a Roman domination function R such that $\sum_{v \in V} R(v) \leq k$? A *Roman domination* of G is a function $R : V \to \{0, 1, 2\}$ such that $\forall v \in V$, if $R(v) = 0$ then there exists $x \in N(v)$ such that $R(x) = 2$.

Parameter: k

FPT (by a $O\left(3.3723^k\, k + |V|^3\right)$ algorithm in [118]; it can improved to a $O^*\left(2^{22.165\sqrt{k}}\right)$ algorithm [118, 11])

Note: See also Roman Domination.

Planar Vertex Cover

Instance: A planar graph $G = (V, E)$; a positive integer k.

Question: Does G have a vertex cover of size at most k? A *vertex cover* is a subset $V' \subseteq V$ such that $\forall\, (v, w) \in E$, $v \in V'$ or $w \in V'$.

Parameter: k

FPT ($O\left(2^{13.0639\sqrt{k}} + kn\right)$ algorithm in [14]; also, $O^*\left(2^{2\sqrt{6k}}\right)$ algorithm in [118]; it does not have a $O^*\left(2^{o\left(\sqrt{k}\right)}\right)$ algorithm unless 3-SAT $\in$ DTIME$(2^{o(n)})$ [49])

Note: See also Vertex Cover.

PLANAR WEIGHTED t-NORMALIZED SATISFIABILITY

Instance: A planar t-normalized boolean expression X; a positive integer k.

Question: Does X have a satisfying truth assignment of weight k?

Parameter: k

Open (reported in [81])

Note: A boolean expression is *t-normalized* if it is of the form product-of-sums-of-products ... of literals with t alternations. Since a 2-normalized boolean expression is in conjunctive normal form, for $t = 2$ the problem coincides with WEIGHTED CNF SATISFIABILITY.

POLYCHROME MATCHING

Instance: An edge-colored graph G with r colors.

Question: Is there a partial matching in G consisting of r edges, with one edge of each color?

Parameter: r

FPT (by the perfect hashing method [81, Exercise 8.3.3])

POLYMATROID RECOGNITION

Instance: A k-polymatroid M; a positive integer k.

Question: Is M hypergraphic?

Parameter: k

Open (reported in [81])

Note: See [209] for definitions.

POLYNOMIAL PRODUCT IDENTITY

Instance: Two sets of k multivariate polynomials p_i and q_i, for $i = 1, \ldots, k$.

Question: Does the identity $\prod_{i=1}^{k} p_i = \prod_{i=1}^{k} q_i$ hold?

Parameter: k

Open (reported in [81])

Polynomially Smooth Number

Instance: An n-bit positive integer N; a positive integer k.

Question: Is N n^k-smooth, i.e., is every prime divisor of N bounded by n^k?

Parameter: k

randomized FPT ([108, 107])

Note: The notion of n^k-smoothness of n-digit numbers arises naturally in the study of polynomial-time complexity. For example, the concept plays a central role in the proof that primality is in UP $\cap$ co-UP.

Positive Weighted Completion Of An Ordering

Instance: A digraph $P = (V, A)$ whose arc relation complies with the axioms of a partial order; a cost function κ mapping $A(D([U(P)]^c))$ into the positive integers (by setting κ to zero for arcs in $A(D(U(P)))$ we can interpret the domain of κ as $V(P) \times V(P)$); a positive integer k.

Question: Is there a selection A' of arcs from $A(D([U(P)^c]))$ such that the transitive closure $(A' \cup A(P))^+$ is a linear order and $\sum_{a \in (A' \cup A(P))^+} \kappa(a) \leq k$?

Parameter: k

FPT ($O\big(1.5175^k + kn^2\big)$ algorithm in [93, 118])

Note: $D(G)$ denotes the digraph obtained from an undirected graph G by replacing every edge (u, v) by the two arcs (u, v) and (v, u). $U(G)$ denotes the undirected graph obtained from a digraph G by putting an edge (u, v) whenever $(u, v) \in A(G)$ or $(v, u) \in A(G)$. The problem is NP-complete by a reduction from One-Sided Crossing Minimization [96].

Power Dominating Set For Almost Trees

Instance: A graph $G = (V, E)$ that is a tree with k edges added; a nonnegative integer k.

Question: Does G have a power dominating set of size at most k? A subset of vertices $M \subseteq V$ is a *power dominating set* if placing a monitoring device (PMU) in every $v \in M$ causes all vertices in V to be observed.

Parameter: k

FPT ($O\big(|V| \cdot 2^{4k \log k}\big)$ algorithm in [137])

Note: The rules for observation are the following: (1) A PMU in a vertex v observes v and all incident edges and neighbors of v; (2) any vertex that is incident to an observed edge is observed; (3) any edge joining two observed vertices is observed; (4) if a vertex is incident to a total of $i > 1$ edges and if $i-1$ of these edges are observed, then all i edges are observed. The problem is NP-complete on general graphs, but it can be solved in linear time on trees [146].

Precedence Constrained Processor Scheduling

Instance: A set T of unit-length tasks; a partial order $\prec$ on T; a positive integer deadline D; a number of processors k.

Question: Is there a map $f : T \to \{1, \ldots, D\}$ such that for all $t, t' \in T$, $t \prec t'$ implies $f(t) < f(t')$, and for all i, $1 \le i \le D$, $\left|f^{-1}(i)\right| \le k$?

Parameter: k

W[2]-hard (reduction from Dominating Set [31])

Note: The general version of this problem is not known to be either NP-complete or in P. [81, Appendix A] cites [33] as a reference for this parameterized problem, but it looks like a mistake.

Profit Vertex Cover

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a subset $C \subseteq V$ with $|E| - |E(G[V \setminus C])| - |C| \ge k$? The *profit of the vertex cover* is the gain of the cover, that is, the size of the edge set that is covered minus the size of the cover.

Parameter: k

FPT ($O\big(k\,|V| + 1.151^k\big)$ algorithm reported in [203])

Proper Intervalizing Colored Graphs

Instance: A vertex-colored graph $G = (V, E)$; a positive integer k.

Question: Is there a $G' \supset G$ which is a proper interval graph and has clique size at most k, and no edge in G' connects two vertices in G with the same color?

Parameter: k

W[t]-hard for all t ([154])

Note: See also Restricted Completion To Proper Interval Graph With Bounded Clique Size.

Pure Implicational Satisfiability Of Fixed F-Depth

Instance: A pure implicational formula X with at most two instances of each proposition and at most k occurrences of the propositional constant **f** (where **f** is always false).

Question: Is X satisfiable?

Parameter: k

FPT (solvable in $O(k^k n^2)$ time [128])

Quantified Antimonotone Circuit Satisfiability

Instance: An integer r; a sequence $s_1, \ldots, s_r$ of pairwise disjoint sets of boolean variables; a decision circuit X with the variables $s_1 \cup \ldots s_r$ as negated inputs and no other inverters; integers $k_1, \ldots, k_r$.

Question: Is it the case that there exists a size k_1 subset t_1 of s_1 such that for every size k_2 subset t_2 of s_2, there exists a size k_3 subset t_3 of s_3 such that … (alternating qualifiers) such that, when the inputs in $t_1 \cup \ldots t_r$ are set to 1 and all other inputs are set to 0, circuit X outputs 1?

Parameter: r, $k_1, \ldots, k_r$

AW[P]-complete (equivalent to Quantified Circuit Satisfiability [2])

Quantified Boolean Antimonotone Formula Satisfiability

Instance: An integer r; a sequence $s_1, \ldots, s_r$ of pairwise disjoint sets of boolean variables; a boolean formula X involving the variables $s_1 \cup \ldots s_r$ in which all variables are negated and no other negation is used; integers $k_1, \ldots, k_r$.

Question: Is it the case that there exists a size k_1 subset t_1 of s_1 such that for every size k_2 subset t_2 of s_2, there exists a size k_3 subset t_3 of s_3 such that … (alternating qualifiers) such that, when the variables in $t_1 \cup \ldots t_r$ are made true and all other variables are made false, formula X is true?

Parameter: r, $k_1, \ldots, k_r$

AW[SAT]-complete (equivalent to QUANTIFIED BOOLEAN FORMULA SATISFIABILITY [2, 81])

QUANTIFIED BOOLEAN FORMULA SATISFIABILITY

Instance: An integer r; a sequence $s_1, \ldots, s_r$ of pairwise disjoint sets of boolean variables; a boolean formula X involving the variables $s_1 \cup \ldots s_r$; integers $k_1, \ldots, k_r$.

Question: Is it the case that there exists a size k_1 subset t_1 of s_1 such that for every size k_2 subset t_2 of s_2, there exists a size k_3 subset t_3 of s_3 such that ... (alternating qualifiers) such that, when the variables in $t_1 \cup \ldots t_r$ are made true and all other variables are made false, formula X is true?

Parameter: r, $k_1, \ldots, k_r$

AW[SAT]-complete (kernel problem for the class AW[SAT])

QUANTIFIED BOOLEAN t-NORMALIZED FORMULA SATISFIABILITY

Instance: An integer r; a sequence $s_1, \ldots, s_r$ of pairwise disjoint sets of boolean variables; a boolean formula X involving the variables $s_1 \cup \ldots s_r$ that consists of t alternating layers of conjunctions and disjunctions with negations applied only to variables (t is a fixed constant); integers $k_1, \ldots, k_r$.

Question: Is it the case that there exists a size k_1 subset t_1 of s_1 such that for every size k_2 subset t_2 of s_2, there exists a size k_3 subset t_3 of s_3 such that ... (alternating qualifiers) such that, when the variables in $t_1 \cup \ldots t_r$ are made true and all other variables are made false, formula X is true?

Parameter: r, $k_1, \ldots, k_r$

AW[$*$]-complete (kernel problem for the class AW[t]; result follows from Trade-off Theorems AW[1] = AW[2] = $\cdots$ = AW[$*$] [1, 2, 85, 86, 81])

Note: See also UNITARY QUANTIFIED BOOLEAN t-NORMALIZED FORMULA SATISFIABILITY.

QUANTIFIED CIRCUIT SATISFIABILITY

Instance: An integer r; a sequence $s_1, \ldots, s_r$ of pairwise disjoint sets of boolean variables; a decision circuit X with the variables $s_1 \cup \ldots s_r$ as inputs;

integers $k_1, \ldots, k_r$.

Question: Is it the case that there exists a size k_1 subset t_1 of s_1 such that for every size k_2 subset t_2 of s_2, there exists a size k_3 subset t_3 of s_3 such that ... (alternating qualifiers) such that, when the inputs in $t_1 \cup \ldots t_r$ are set to 1 and all other inputs are set to 0, circuit X outputs 1?

Parameter: r, $k_1, \ldots, k_r$

AW[P]-complete (kernel problem for the class AW[P])

QUANTIFIED MONOTONE CIRCUIT SATISFIABILITY

Instance: An integer r; a sequence $s_1, \ldots, s_r$ of pairwise disjoint sets of boolean variables; a decision circuit X with the variables $s_1 \cup \ldots s_r$ as inputs and no inverters; integers $k_1, \ldots, k_r$.

Question: Is it the case that there exists a size k_1 subset t_1 of s_1 such that for every size k_2 subset t_2 of s_2, there exists a size k_3 subset t_3 of s_3 such that ... (alternating qualifiers) such that, when the inputs in $t_1 \cup \ldots t_r$ are set to 1 and all other inputs are set to 0, circuit X outputs 1?

Parameter: r, $k_1, \ldots, k_r$

AW[P]-complete (membership: reduction to QUANTIFIED CIRCUIT SATISFIABILITY [2]; hardness: reduction from MINIMUM AXIOM SET [83, 82])

QUANTIFIED BOOLEAN MONOTONE FORMULA SATISFIABILITY

Instance: An integer r; a sequence $s_1, \ldots, s_r$ of pairwise disjoint sets of boolean variables; a boolean monotone formula X involving the variables $s_1 \cup \ldots s_r$; integers $k_1, \ldots, k_r$.

Question: Is it the case that there exists a size k_1 subset t_1 of s_1 such that for every size k_2 subset t_2 of s_2, there exists a size k_3 subset t_3 of s_3 such that ... (alternating qualifiers) such that, when the variables in $t_1 \cup \ldots t_r$ are made true and all other variables are made false, formula X is true?

Parameter: r, $k_1, \ldots, k_r$

AW[SAT]-complete (equivalent to QUANTIFIED BOOLEAN FORMULA SATISFIABILITY [2, 81])

Reachibility Distance For Vector Addition Systems

Instance: A set T of m integer-valued vectors $T = \{x^i = (x_1^i, \ldots, x_n^i) : 1 \le i \le m\}$; a non-negative starting vector $s = (s_1, \ldots, s_n)$; a non-negative target vector $t = (t_1, \ldots, t_n)$; a positive integer k.

Question: Is there a choice of k indices $i_1, \ldots, i_k$, $1 \le i_j \le m$ for $j = 1, \ldots, k$ such that $t = s + \sum_{j=1}^{k} x^{i_j}$ and such that every intermediate sum is non-negative in each component, that is, $s_r + \sum_{j=1}^{q} x_r^{i_j} \ge 0$ for $q = 1, \ldots, k$ and $r = 1, \ldots, n$?

Parameter: k

W[1]-hard (reduction from Clique [83, 82])

Note: Also known as Reachibility Distance for Petri Nets.

Real Weighted Vertex Cover

Instance: A graph $G = (V, E)$; a weight function $\omega : V \to [1, \infty)$; $k \in \mathbb{R}^+$.

Question: Is there a subset of vertices $V' \subseteq V$ of weight at most k such that every edge of G has at least one endpoint in V'?

Parameter: k

FPT (solvable in time $O\big(1.3954^k + kn\big)$ [174])

Note: The general problem having an arbitrary real function $\omega : V \to \mathbb{R}^+$ is NP-complete for any fixed k, thus it cannot be fixed-parameter tractable unless P = NP [172]. See also Vertex Cover and Integer Weighted Vertex Cover.

Rectilinear Picture Compression

Instance: A boolean $n \times n$ matrix; a positive integer k.

Question: Are there k rectangles that cover all the 1's?

Parameter: k

FPT (by the reduction to problem kernel method [81, Exercise 3.2.6])

Red-Blue Dominating Set

Instance: A graph $G = (V, E)$ with V partitioned in $V_{\text{red}} \cup V_{\text{blue}}$; a positive integer k.

Question: Is there a subset $D \subseteq V_{\text{red}}$ with $|D| \leq k$ and $V_{\text{blue}} \subseteq N(D)$ (that is, $\forall\, v \in V_{\text{blue}}, \exists\, w \in D$ such that $(v, w) \in E$)?

Parameter: k

W[2]-complete (equivalent to Hitting Set [118])

Note: See also Planar Red-Blue Dominating Set.

Restricted Alternating Hitting Set

Instance: A collection C of subsets of a set B with $|c| \leq k_1$ for all $c \in C$; an integer k_2.

Question: Does player one have a forced win in no more than k_2 moves in the following game played on C and B? Players alternate choosing a new element of B until, for each $c \in C$, some member of c has been chosen. The player whoose choice causes this to happen loses.

Parameter: k_1, k_2

FPT (solvable in $O(f(k_1, k_2) \cdot n)$ time [2])

Restricted Completion To Proper Interval Graph With Bounded Clique Size

Instance: A graph $G = (V, E)$; a set $E' \subseteq V \times V - E$ of "prohibited" edges; a positive integer k.

Question: Is there a $G' \supset G$ which is a proper interval graph and has clique size at most k, and G' has no edges from E'?

Parameter: k

W[t]-hard for all t ([154, 155]; it remains W[t]-hard even when $E' = \emptyset$ [154])

Note: See Proper Intervalizing Colored Graphs for a restricted version of this problem.

Restricted Valence Isomorphism

Instance: Two graphs $G = (V, E)$ and $H = (V', E')$; a positive integer k.

Question: Are G and H isomorphic graphs such that the valencies of the vertices of both G and H are bounded by k?

Parameter: k

Open (reported in [81]; if this problem is W[1]-hard, the classic GRAPH ISOMORPHISM problem is not in P unless FPT = W[1].)

ROMAN DOMINATION

Instance: A planar graph $G = (V, E)$; a positive integer k.

Question: Is there a Roman domination function R such that $\sum_{v \in V} R(v) \leq k$? A *Roman domination* of G is a function $R : V \to \{0, 1, 2\}$ such that $\forall v \in V$, if $R(v) = 0$ then there exists $x \in N(v)$ such that $R(x) = 2$.

Parameter: k

W[2]-complete (membership: by a reduction to SHORT MULTI-TAPE NONDETERMINISTIC TURING MACHINE COMPUTATION [118]; hardness: by a reduction from RED-BLUE DOMINATING SET restricted to bipartite graphs [118])

Parameter: treewidth of G

FPT ($O\big(5^{\mathrm{tw}(G)}\,|V|\big)$ algorithm in [118])

Note: See also PLANAR ROMAN DOMINATION.

RUSH HOUR PUZZLE

Instance: A tuple (C, S, p^0, d, Z), where C is a finite set (representing the *cars*), $S : C \to 2^{\mathbb{R}^2}$ defines the *car shapes* (always axes-parallel open rectangles contained in the first quadrant of the plane and with the origin as a corner), $p^0 : C \to \mathbb{R}^2$ defines the initial *car positions*, $d : C \to \{(1,0), (0,1), (0,0)\}$ defines the *car directional vectors*, and $Z : C \to 2^{\mathbb{R}}$ defines the set of the Rush Hour goals; a positive integer m.

Question: Is there a sequence of $m+1$ configurations $u : C \to \mathbb{R}$ that are *legal* (that is, $(p^0(c) + u(c)d(c) + S(c)) \cap (p^0(c') + u(c')d(c') + S(c')) = \emptyset$ for all $c, c' \in C$ with $c \neq c'$) and such that the first configuration consists of the cars in the initial position, the last configuration consists of cars in goal positions, and each pair of successive configurations is connected via a *legal move*, that is, via an operation that adds to (or subtracts from) the position of a car a multiple of its directional vector?

Parameter: $|C|$

FPT (exhaustive $O\big((2k^3)^{2k}p(n)\big)$ algorithm (where $p(n)$ is a polynomial and the $n \times n$ square initially includes all cars) that generates and analyzes the search space of discrete configurations [119, 118])

Parameter: m

FPT ($2^{O(m^2 \cdot 3^m)}p(n)$ algorithm in [119, 118])

Note: The classical version of this problem is PSPACE-complete [121].

Shadow Independent Set

Instance: A forest $F_k(n) = \{T_i : 1 \leq i \leq k\}$ with n nodes consisting of k trees T_i rooted at r_i and with leaf sets L_i; a partial map $\sigma : \bigcup_i L_i \to \bigcup_i V(T_i) \backslash \{r_i\}$ such that $\sigma(L_i) \cap V(T_i) = \emptyset$ and domain$(\sigma) \cap L_i \neq \emptyset$ $(1 \leq i \leq k)$.

Question: Is there a set $S = \{x_i \in L_i : 1 \leq i \leq k\}$ containing exactly one leaf of each tree such that for every $x_i, x_j \in S$, $x_j \notin s_{\sigma(x_i)}$? $s_{\sigma(x_i)}$ is the *shadow* of x_j and includes all leaves of the subtree rooted at $\sigma(x_i)$.

Parameter: k

FPT ($O\big(n^2 k^k\big)$ algorithm in [128]; $O\big(n^3 3^k\big)$ dynamic programming algorithm in [148])

Note: Equivalent to the falsifiability problem for pure implicational Boolean formulas over n variables with k occurrences of the constant *false* [147].

Search Number

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Are k searchers sufficient to ensure the capture of a fugitive who is free to move with arbitrary speed along the edges of G?

Parameter: k

FPT ([111, 115]; solvable in $O(f(k) \cdot n)$ time [23])

Semigroup Embedding

Instance: A semigroup $(S, \cdot)$; a semigroup $(H, \times)$.

Question: Can H be embedded into S?

Parameter: H

W[1]-complete (membership: direct proof [81, Exercise 10.0.3]; hardness: reduction from Clique [38, 184], also in [81, Exercise 10.0.3])

Semilattice Embedding

Instance: A semilattice L; a semilattice H.

Question: Can H be embedded into L?

Parameter: H

W[1]-complete (membership: direct proof [81, Exercise 10.0.3]; hardness: reduction from Clique [38, 184], also in [81, Exercise 10.0.3])

Separating Into Components

Instance: A graph $G = (V, E)$; integers k and ℓ.

Question: Is there a set $S \subseteq V$ of size k such that $G \setminus S$ has at least ℓ connected components?

Parameter: k, ℓ

W[1]-hard (reduction from Clique [167])

Note: See also Separating Connected Vertices and Separating Vertices.

Separating Connected Vertices

Instance: A graph $G = (V, E)$; integers k and ℓ.

Question: Is there a partition $V = X \cup S \cup Y$ such that $G[X]$ is a connected subgraph of G, $|X| = \ell$, $|S| \leq k$ and there is no edge between X and Y?

Parameter: k

W[1]-hard (reduction from Clique [167])

Parameter: ℓ

W[1]-hard (reduction from Clique [167])

Parameter: k, ℓ

FPT (algorithm based on perfect hash functions in [167])

Note: See also Separating Into Components and Separating Vertices.

Separating Vertices

Instance: A graph $G = (V, E)$; integers k and ℓ.

Question: Is there a partition $V = X \cup S \cup Y$ such that $|X| = \ell$, $|S| \leq k$ and there is no edge between X and Y?

Parameter: k, ℓ

W[1]-hard (reduction from Clique [167])

Parameter: k, ℓ, $\max_{v \in V} \deg(v)$

FPT (algorithm based on perfect hash functions in [167])

Note: See also Separating Into Components and Separating Connected Vertices.

Set Basis

Instance: A collection C of subsets of a finite set S; a positive integer k.

Question: Is there a collection B of subsets of S with $|B| = k$ such that, for every set $A \in C$, there is a subcollection of B whose union is exactly A?

Parameter: k

FPT (by the reduction to problem kernel method [81, Exercise 3.2.3])

Set Cover

Instance: A finite family of sets $S = S_1, \ldots, S_n$; a positive integer k.

Question: Is there a subset $R \subseteq S$ with $|R| \leq k$ whose union is all elements in the union of S?

Parameter: k

W[2]-complete (hardness: reduction from Dominating Set [181, 210])

Note: Often in the literature the name "Set Cover" denotes the Hitting Set problem. The general problem is NP-complete even if each element occurs in at most two sets of the family S [178].

Set Packing

Instance: A finite family of sets $S = S_1, \ldots, S_n$; a positive integer k.

Question: Does S contain a subset R of k mutually disjoint sets?

Parameter: k

W[1]-complete (hardness: reduction from Independent Set [16, 210])

Set Splitting

Instance: A collection $\mathcal{F}$ of subsets of a finite set X; a positive integer k.

Question: Is there a subfamily $\mathcal{F}' \subseteq \mathcal{F}$ with $|\mathcal{F}| \geq k$ and a partition of X into disjoint subsets X_0 and X_1 such that for every $S \in \mathcal{F}'$, $S \cap X_0 \neq \emptyset$ and $S \cap X_1 \neq \emptyset$?

Parameter: k

FPT (tractability results in [67, 68]; current best algorithm is obtained by a reduction to a problem kernel of size $2k$, which yields a $O^*(2.6494^k)$ algorithm [163, 164])

Note: Equivalent to Bipartite Colorful Neighborhood [163].

Short Cheap Tour

Instance: A graph $G = (V, E)$; a weight function $w : E \to \mathbb{Z}$; positive integers S and k.

Question: Is there a tour through at least k nodes of G of cost at most S?

Parameter: k

FPT ([183])

Note: See also Exact Cheap Tour.

Short Circuit Satisfiability

Instance: A boolean circuit C with n gates, at most $k \log n$ inputs, and one output; a positive integer k.

Question: Is there a setting of the inputs that cause C to output 1?

Parameter: k

W[P]-complete (hardness: reduction from Weighted Circuit Satisfiability [2, 3])

Short Context-Free Derivation

Instance: A context-free grammar $G = (N, \Sigma, \Pi, S)$ where N is a finite set of "nonterminal symbols", Σ is a finite set of "terminal symbols", Π is a finite set of "production rules" of the form $(A \to \beta)$ with $A \in N$ and $\beta \in (N \cup \Sigma)^*$, and $S \in N$ is the "start symbol"; a word $x \in \Sigma^*$; a positive integer k.

Question: Is there a G-derivation of x of length at most k? That is, is there a sequence of words $x_0, \ldots, x_k$ with $x_i \in (N \cup \Sigma)^*$ for $i = 0, \ldots, k$, that satisfies the requirements: (1) $x_0 = S$, (2) $x_k = x$, and (3) for each $i = 1, \ldots, k$ there is a production rule $(B \to \gamma) \in \Pi$ such that $x_{i-1} = \alpha B \delta$ and $x_i = \alpha\gamma\delta$?

Parameter: k

FPT ([81, Exercise 11.0.1])

Note: See also the problems Short Context-Sensitive Derivation and Short Grammar Derivation.

Short Context-Sensitive Derivation

Instance: A context-sensitive grammar $G = (N, \Sigma, \Pi, S)$ where N is a finite set of "nonterminal symbols", Σ is a finite set of "terminal symbols", $S \in N$ is the "start symbol", and Π is a finite set of "production rules" of the form $(\alpha \to \beta)$ with $\alpha, \beta \in (N \cup \Sigma)^*$, $\alpha, \beta \neq \epsilon$, and $|\alpha| \leq |\beta|$; a word $x \in \Sigma^*$; a positive integer k.

Question: Is there a G-derivation of x of length at most k? That is, is there a sequence of words $x_0, \ldots, x_k$ with $x_i \in (N \cup \Sigma)^*$ for $i = 0, \ldots, k$, that satisfies the requirements: (1) $x_0 = S$, (2) $x_k = x$, and (3) for each $i = 1, \ldots, k$ there is a production rule $(\beta \to \gamma) \in \Pi$ such that $x_{i-1} = \alpha\beta\delta$ and $x_i = \alpha\gamma\delta$?

Parameter: k

W[1]-complete (membership: reduction to Short Nondeterministic Turing Machine Computation [48]; hardness: reduction from Clique [83, 82])

Note: The general version of this problem is PSPACE-complete by a reduction from Linear Space Bounded Automaton Acceptance. See also Short Context-Free Derivation and Short Grammar Derivation.

Short Deterministic Turing Machine Computation

Instance: A single-tape, single-head deterministic Turing machine $M = (\Sigma, Q, \Delta)$, where Σ is an alphabet, Q is a set of internal states, and Δ

is a set of transitions; a word $x \in \Sigma^*$; a positive integer k.

Question: Does $M(x)$ reach the final accepting state in at most k steps?

Parameter: k

FPT (by simulating M with a Universal Turing Machine [50, 54])

Note: The general version of this problem is undecidable.

SHORT 3-DIMENSIONAL MATCHING

Instance: Set $M \subseteq X \times Y \times Z$ where X, Y and Z are disjoint sets having the same number of elements; a positive integer k.

Question: Does there exist a set $M' \subseteq M$ such that $|M'| = k$ and for all $(x, y, z), (x', y', z') \in M'$ with $(x, y, z) \neq (x', y', z')$, we have $x \neq x'$, $y \neq y'$ and $z \neq z'$?

Parameter: k

FPT ([81, Exercise 8.2.3])

SHORT FORMULA SATISFIABILITY

Instance: A boolean formula φ of n variables; a list of at most $k \log n$ variables of φ; a positive integer k.

Question: Is there any setting of the distinguished variables that causes φ to unravel?

Parameter: k

W[P]-complete (hardness: reduction from SHORT CIRCUIT SATISFIABILITY [2])

SHORT GENERALIZED GEOGRAPHY

Instance: A directed graph $D = (V, A)$; a vertex $v_0 \in V$; a positive integer k.

Question: Does player one have a winning strategy in k moves for the following game? Players alternately choose a new arc from A. The first arc chosen must have its tail at v_0, and each subsequently chosen arc must have its tail at the vertex that was the head of the previous arc. The first player unable to choose a new arc loses. Player one plays first.

Parameter: k

AW[*]-complete (membership: direct proof [2]; hardness: reduction from Unitary Quantified Boolean t-Normalized Formula Satisfiability [1, 2])

Short Generalized Hex

Instance: A graph $G = (V, E)$; two distinguished vertices $v_1, v_2 \in V$; a positive integer k.

Question: Does player one have a winning strategy of at most k moves in the following game? Player one plays with white pebbles and player two with black ones. Pebbles are placed on nondistinguished vertices alternately by player one, then player two. Player one wins if he can construct a path of white vertices from v_1 to v_2.

Parameter: k

Open (reported in [81]; candidate for AW[*]-completeness)

Short Grammar Derivation

Instance: A context-sensitive grammar $G = (N, \Sigma, \Pi, S)$ where N is a finite set of "nonterminal symbols", Σ is a finite set of "terminal symbols", Π is a finite set of "production rules" of the form $(\alpha \to \beta)$ with $\alpha, \beta \in (N \cup \Sigma)^*$, and $S \in N$ is the "start symbol"; a word $x \in \Sigma^*$; a positive integer k.

Question: Is there a G-derivation of x of length at most k? That is, is there a sequence of words $x_0, \ldots, x_k$ with $x_i \in (N \cup \Sigma)^*$ for $i = 0, \ldots, k$, that satisfies the requirements: (1) $x_0 = S$, (2) $x_k = x$, and (3) for each $i = 1, \ldots, k$ there is a production rule $(\beta \to \gamma) \in \Pi$ such that $x_{i-1} = \alpha\beta\delta$ and $x_i = \alpha\gamma\delta$?

Parameter: k

W[1]-complete (membership: reduction to Short Nondeterministic Turing Machine Computation [48]; hardness: reduction from Clique [48])

Note: The general version of this problem is PSPACE-complete by a reduction from Linear Space Bounded Automaton Acceptance. See also Short Context-Free Derivation and Short Context-Sensitive Derivation.

Short Multi-Head Nondeterministic Turing Machine Computation

Instance: A nondeterministic Turing machine $M = (\Sigma, Q, \Delta)$ with t tapes

($t \geq 2$) and h heads on each tape ($h \geq 2$), where Σ is an alphabet, Q is a set of internal states, and Δ is a set of transitions; a word $x \in \Sigma^*$; a positive integer k.

Question: Is there a computation of M on input x that reaches the final accepting state in at most k steps?

Parameter: k

W[2]-hard (trivial reduction from Short Multi-Tape Nondeterministic Turing Machine Computation [50, 54]; it remains W[2]-hard if M has empty input, one non-final internal state, and one tape [50, 54]; if M is *total*, that is, Δ includes at least one transition for every possible combination of scanned symbols and internal states, the problem is W[1]-complete [50, 54])

Parameter: k, t, h

W[1]-complete (equivalent to Short Nondeterministic Turing Machine Computation [50, 54]; it remains W[1]-complete if M has empty input, one tape, two heads, and one non-final internal state [50, 54])

Parameter: k, t, h, $|\Sigma|$

FPT (by exhaustively checking all global configurations of $M(x)$ [50, 54])

Note: The general version of this problem is undecidable.

Short Multi-Tape Nondeterministic Turing Machine Computation

Instance: A single-head nondeterministic Turing machine $M = (\Sigma, Q, \Delta)$ with t tapes ($t \geq 2$), where Σ is an alphabet, Q is a set of internal states, and Δ is a set of transitions; a word $x \in \Sigma^*$; a positive integer k.

Question: Is there a computation of M on input x that reaches the final accepting state in at most k steps?

Parameter: k

W[2]-complete (membership: direct proof [53]; hardness: reduction from Dominating Set [50, 54]; it remains W[2]-complete if M has empty input, one non-final internal state, and two non-blank symbols [50, 54]; if M is *total*, that is, Δ includes at least one transition for every possible combination of scanned symbols and internal states, the problem is W[1]-complete [50, 54])

Parameter: k, t

W[1]-complete (equivalent to Short Nondeterministic Turing Machine Computation [50, 54]; it remains W[1]-complete if M has just two writable tapes, one non-final internal state, and empty input [50, 54])

Parameter: k, t, $|\Sigma|$

FPT (by exhaustively checking all global configurations of $M(x)$ [50, 54])

Note: The general version of this problem is undecidable.

Short Node Kayles

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does player one have a winning strategy in k moves for the following game? Players pebble a vertex not adjacent to any pebbled vertex. The first player with no play loses. Player one plays first.

Parameter: k

AW[$*$]-complete (membership: direct proof [2] and [81, Exercise 14.0.2]; hardness: reduction from Unitary Quantified Boolean t-Normalized Formula Satisfiability [2] and [81, Exercise 14.0.2])

Short Nondeterministic Turing Machine Computation

Instance: A single-tape, single-head nondeterministic Turing machine $M = (\Sigma, Q, \Delta)$, where Σ is an alphabet, Q is a set of internal states, and Δ is a set of transitions; a word $x \in \Sigma^*$; a positive integer k.

Question: Is there a computation of M on input x that reaches the final accepting state in at most k s teps?

Parameter: k

W[1]-complete (membership: direct proof [48, 83, 82]; hardness: reduction from CLIQUE [48, 83, 82]; it remains W[1]-complete if the input word x is the null string; it belongs to FPT if the maximum number of nondeterministic transition possibilities out of an internal state is fixed.)

Parameter: k, $|\Sigma|$

FPT (by exhaustively checking all global configurations of $M(x)$ [50, 54])

Parameter: k, $|Q|$

FPT (by reducing to the same problem parameterized by k and $|\Sigma|$ [50, 54])

Note: The general version of this problem is undecidable.

SHORT PHONOLOGICAL SEGMENT DECODING

Instance: A simplified segmental grammar $s = (F, S, D, R, c_p, C)$ such that the number of mutually exclusive rule sets in R, $|R_{\text{m.e.}}|$, is at most k; a string $s \in S^+$.

Question: Is there a string $u \in D$ such that $g(u) = s$?

Parameter: k

W[t]-hard for all t, in W[P] (membership: direct proof ? [77]; hardness: reduction from WEIGHTED t-NORMALIZED SATISFIABILITY [77]; the same proofs also implies the W[t]-hardness and membership in W[P] of SHORT PHONOLOGICAL SEGMENT ENCODING)

Note: See [77] and [191] for the definition of simplified segmental grammar.

SHORT POST CORRESPONDENCE

Instance: A Post system Π; a positive integer k.

Question: Is there a length k solution for Π?

Parameter: k

W[1]-complete (membership: reduction to SHORT NONDETERMINISTIC TURING MACHINE COMPUTATION [48]; hardness: reduction from SHORT GRAMMAR DERIVATION [48])

Note: The general version is undecidable.

Short Semi-Thue Process

Instance: A semi-Thue process Π consisting of a finite alphabet A together with a set of production rules $g \to \overline{g}$, where $g \in A$; words $w, v \in A^*$; a positive integer k.

Question: Does $w \Rightarrow_{\Pi}^{*} v$ in no more than k steps?

Parameter: k

W[1]-complete (membership: reduction to Short Post Correspondence [81]; hardness: reduction from Clique [81])

Note: The general version is undecidable.

Shortest Common Supersequence

Instance: An alphabet Σ; a set of strings $\{r_1, \ldots, r_k\}$ formed over alphabet Σ; a positive integer λ.

Question: Does there exist a string $s \in \Sigma^*$ of length at most λ such that s is a supersequence of each string r_i, $1 \leq i \leq k$? (A string s is a *supersequence* of a string r if we can delete some characters in s such that the remaining string is equal to r.)

Parameter: k, $|\Sigma|$

W[t]-hard for all t ([144, 143])

Parameter: λ

FPT (reported by [105])

Note: See also Fixed Alphabet Shortest Common Supersequence.

Shortest Vector

Instance: A basis $X = \{x_1, x_2, \ldots, x_n\} \subset \mathbb{Z}^n$ for a lattice Ł; a positive integer k.

Question: Is there a non-zero vector $x \in$ Ł such that $\|x\|^2 \leq k$?

Parameter: k

Open (reported in [81])

Small Herbrand Model

Instance: A finite propositional logic program P; a positive integer k.

Question: Does there exist an Herbrand model M for P of size no more than k?

Parameter: k

W[2]-complete ([51])

Note: The problem remains W[2]-complete if the Herbrand model is required to be *minimal*, that is, every proper subset of M is not an Herbrand model for P [51]. See also SMALL STABLE MODEL.

SMALL MINIMUM DEGREE FOUR SUBGRAPH

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a subgraph of G of minimum degree at least 4 and of cardinality at most k?

Parameter: k

W[1]-complete ([210])

SMALL PRIME DIVISOR

Instance: An n-bit positive integer N; a positive integer k.

Question: Does N have a nontrivial divisor less than n^k?

Parameter: k

randomized FPT ([108, 107])

SMALL STABLE MODEL

Instance: A finite propositional logic program P; a non-negative integer k.

Question: Does there exist a stable model for P of size no more than k?

Parameter: k

W[2]-complete (membership: reduction to SHORT MULTI-TAPE NONDETERMINISTIC TURING MACHINE COMPUTATION [51]; hardness: reduction from WEIGHTED CNF SATISFIABILITY [208])

Note: A model M of a logic program P is a *stable model* if M coincides with the minimal Herbrand model M for the the logic program P_M obtained from P by deleting (i) each rule having a negative literal $\neg v$ in its body with $v \in M$ and (ii) all negative literals in the bodies of the remaining rules [131]. See also Large Stable Model and Small Herbrand Model.

Spare Allocation

Instance: An $n \times m$ binary matrix A representing an erroneous chip, with $A[i,j] = 1$ if and only if the chip is faulty at position $[i,j]$; positive integer k_1, k_2.

Question: Is there a *reconfiguration strategy*—i.e., a description of which rows and columns of A have to be replaced by spares—that repairs all faults and uses at most k_1 spare rows and at most k_2 spare columns?

Parameter: k_1, k_2

FPT (equivalent to Constraint Bipartite Vertex Cover [159])

Square Tiling

Instance: A set C of "colors"; a collection $T \subseteq C^4$ of "tiles" (where $\langle a,b,c,d\rangle$ denotes a tile whose top, right, bottom, and left sides are colored a, b, c, and d, respectively); a positive integer k.

Question: Is there a tiling of a $k \times k$ square using the tiles in T, i.e., an assigment of a tile $f(i,j) \in T$ to each ordered pair i,j, $1 \leq i \leq k$, $1 \leq j \leq k$, such that (1) if $f(i,j) = \langle a,b,c,d\rangle$ and $f(i+1,j) = \langle a',b',c',d'\rangle$, then $a = c'$, and (2) if $f(i,j) = \langle a,b,c,d\rangle$ and $f(i,j+1) = \langle a',b',c',d'\rangle$, then $b = d'$?

Parameter: k

W[1]-complete (membership: reduction to Short Nondeterministic Turing Machine Computation [81]; hardness: reduction from Clique [81])

s-Star Packing

Instance: A graph $G = (V,E)$; a positive integer k.

Question: Are there k vertex disjoint instances of $K_{1,s}$ $(s \geq 3)$ in G?

Parameter: k

FPT (by a reduction to problem kernel of size $k(s^3+ks^2+ks+1)$ yielding a $O^*\big(2^{O(k \log k)}\big)$ algorithm [186])

Note: See also 3-Path Packing, Triangle Packing, and Graph Packing.

Steiner Tree

Instance: A graph $G = (V, E)$; a set S of at most k vertices in V; an integer m.

Question: Is there a set of vertices $T \subseteq V - S$ such that $|T| \leq m$ and $G[S \cup T]$ is connected?

Parameter: m

W[2]-complete (membership: reduction to Short Multi-Tape Nondeterministic Turing Machine Computation [53]; hardness: reduction from Dominating Set [36])

Parameter: k

FPT ([90]; solvable in time $O(3^k n + 2^k n^2 + n^3)$ by the Dreyfus-Wagner algorithm [210])

Steiner Tree In HyperCubes

Instance: Binary sequences $X_1, \ldots, X_k$, where each X_i has length q; a positive integer M encoded in binary.

Question: Is there a subgraph S of the q-dimensional binary hypercube that includes the vertices $X_1, \ldots, X_k$, such that S has at most M edges? Two vertices are adjacent if and only if the corresponding vectors differ in a single component.

Parameter: k

FPT (by the reduction to problem kernel method (H. T. Wareham) [81, Exercise 3.2.11])

Subset Product

Instance: A set of integers $X = \{x_1, \ldots, x_n\}$; integers a and m; a positive integer k.

Question: Is there a subset $X' \subseteq X$ of cardinality k such that the product of the integers in X' is congruent to a mod m?

Parameter: k

W[1]-hard (reduction from Perfect Code [108, 109])

Subset Sum

Instance: A set of integers $X = \{x_1, \ldots, x_n\}$; an integer s; a positive integer k.

Question: Is there a subset $X' \subseteq X$ of cardinality k such that the sum of the integers in X' equals s?

Parameter: k

W[1]-hard, in W[P] (membership: [109]; hardness: reduction from Perfect Code [78])

Theta Series

Instance: A basis $X = \{x_1, \ldots, x_n\} \subset \mathbb{Z}^n$ for a lattice Ł; a positive integer k.

Question: Is there a vector $x \in$ Ł such that $\|x\|^2 = k$?

Parameter: k

W[1]-hard (reduction from Perfect Code [89])

Note: See also Nearest Vector.

Threshold Starting Set

Instance: A directed graph $D = (V, A)$; a positive integer k.

Question: Does G have a starting set of size k? A *starting set* is a set of vertices $V' \subseteq V$ with the property that if we begin with a pebble on each of the vertices in V' and subsequently place pebbles on any vertex having at least t incoming arcs from pebbled vertices, then eventually every vertex of the graph is pebbled.

Parameter: k

W[P]-complete (hardness: reduction from Weighted Monotone Circuit Satisfiability [2])

Note: t is a fixed constant?!

t-Threshold Stable Set

Instance: A directed graph $G = (V, A)$; a positive integer k.

Question: Does G have a t-threshold stable set of size k? A *t-threshold stable set* is a set of vertices $V' \subseteq V$ such that for every vertex v of $V - V'$, there are fewer than t vertices $u \in V'$ with $(u, v) \in A$.

Parameter: k

W[1]-complete (hardness: reduction from INDEPENDENT SET [81])

Note: The problem is included in [81, Appendix A] with a reference to the book itself, but I was not able to find other references to the problem in the book.

TOPOLOGICAL BANDWIDTH

Instance: A connected graph $G = (V, E)$; a positive integer k.

Question: Is the topological bandwidth of G at most k? The *topological bandwidth* of a graph G is the minimum bandwidth over all graphs obtainable by subdividing the edges of G.

Parameter: k

W[t]-hard for all t (reduction from BANDWIDTH [33])

TOPOLOGICAL CONTAINMENT

Instance: Graphs G and H.

Question: Is H topologically contained in G?

Parameter: H

Open (reported in [81])

TOURNAMENT DOMINATING SET

Instance: A tournament $G = (V, A)$, that is, a directed graph such that for all $x, y \in V$, exactly one of (x, y) and (y, x) is in A; a positive integer k.

Question: Does G have a dominating set of cardinality at most k?

Parameter: k

W[2]-complete (membership is easy; hardness: reduction from DOMINATING SET [79] [80] [81, Exercise 12.0.3])

Note: The general version of this problem is LOGSNP-complete.

Tree-like Weighted Set Cover with Bounded Occurrence

Instance: A base set U; a tree-like collection $C = \{c_1, c_2, \ldots c_m\}$ of subsets of U, where each element of U can be in at most d subsets for a fixed $d \geq 1$; a weight function $w : C \rightarrow \mathbb{R}^+$; a positive integer q.

Question: Is there a subset $C' \subseteq C$ having weight $\leq q$ that covers all elements in U, i.e., $\cup_{c \in C'} c = U$?

Parameter: number of leaves of the subset tree T for C

FPT ($O\left(2^{dk^2} \cdot m^2 n\right)$ algorithm, where $n = |U|$ and k denotes the number of leaves of the subset tree [137])

Note: A collection C of subsets is *tree-like* if we can organize the subsets in C in an unrooted tree T such that every subset one-to-one corresponds to a node of T and, for each element $u \in U$, all nodes in T corresponding to the subsets containing u induce a subtree of T. The general problem is NP-complete for $d \geq 3$ even if the underlying subset tree is a star [138]; it can be solved in $O\left(m^2 n\right)$ time if the underlying subset tree is a path [138].

Treewidth

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is the treewidth of G no more than k?

Parameter: k

FPT (Bodlaender's Theorem [25], yielding an $O(f(k) \cdot n)$ algorithm)

Triangle Edge Deletion

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there an edge set $C \subseteq E$ with $|C| \leq k$ whose removal produces a graph without triangles as vertex-induced subgraphs?

Parameter: k

FPT ([133]; best current algorithm is obtained by reducing to Hitting Set For Size d Sets with $d = 3$ and has time complexity in $O\left(2.1788^k + |E|^3\right)$ [118])

Triangle Vertex Deletion

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there an vertex set $C \subseteq V$ with $|C| \leq k$ whose removal produces a graph without triangles as vertex-induced subgraphs?

Parameter: k

FPT ([133]; best current algorithm is obtained by reducing to Hitting Set For Size d Sets with $d = 3$ and has time complexity in $O\left(2.1788^k + |V|^3\right)$ [118])

Triangle Packing

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Are there at least k vertex disjoint instances of K_3 in G?

Parameter: k

FPT (reduction to problem kernel of size $O(k^3)$ [106])

Note: See also 3-Path Packing, s-Star Packing, and Graph Packing.

Triangulating Colored Graphs

Instance: A graph $G = (V, E)$; a vertex coloring $c : V \to \{1, \ldots, k\}$.

Question: Does there exist a supergraph $G' = (V', E')$ where $E \subseteq E'$, G' is properly colored by c, and G' is triangulated?

Parameter: k

W[t]-hard for all t (reduction from Longest Common Subsequence parameterized by k [33])

Two-Layer Planarization

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a set $C \subseteq E$, $|C| \leq k$, whose removal makes the graph biplanar?

Parameter: k

FPT ($O(k^2 \cdot 5.1926^k + |G|)$ algorithm in [92, 118])

Note: A bipartite graph $G = (V_1, V_2, E)$ is *biplanar* if the vertices can be placed on two parallel lines L_1 and L_2 in the plane (where the vertices from V_i are placed on L_i) such that there are no edge crossings when edges are drawn as straight-line segments. A graph $G = (V, E)$ is biplanar if there is a bipartition $V = V_1 \cup V_2$ of its edge set (i.e., $E(G[V_1]) = E(G[V_2]) = \emptyset$) such that (V_1, V_2, E) is biplanar. See also One-Layer Planarization and t-Layer Planarization.

Two-Tree Crossing Minimization

Instance: A two-tree (T_1, T_2) with leaf labels Λ; a positive integer k.

Question: Can (T_1, T_2) be drawn on the plane in such a way that the corresponding leaves are aligned in two adjacent layers with at most k crossings?

Parameter: k

FPT ($O^*\left(c^k\right)$ algorithm where c is a huge constant [118])

Note: A *two-tree* is a pair of rooted binary trees with perfect matching between corresponding leaves of the two trees, where the correspondence is given by an appropriate labeling such that only leaves with the same label are matched. If the drawing must respect a fixed ordering of the vertices in one of the trees, the problem can be solved in time $O\left(n \log^2 n\right)$, where n is the number of leaves [118].

Uniform Emulation For A Directed Graph

Instance: A directed acyclic graph $G = (V, E)$; a positive integer k.

Question: Does there exist a function $f : V \to \{1, \ldots, |V|/k\}$ such that $(u, v) \in E$ implies $f(u) - f(v) \in \{0, 1\}$ and for all $i = 1, \ldots, |V|/k$, $\left|f^{-1}(i)\right| \leq k$?

Parameter: k

W[t]-hard for all t (reduction from Weighted Monotone t-Normalized Satisfiability [33, 32])

Uniform Emulation On A Path

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does there exist a function $f : V \to \{1, \ldots, |V|/k\}$ such that $(u, v) \in E$ implies $|f(u) - f(v)| \leq 1$ and for all $i = 1, \ldots, |V|/k$, $\left|f^{-1}(i)\right| \leq k$?

Parameter: k

W[t]-hard for all t (reduction from Weighted Monotone t-Normalized Satisfiability [33]; remains W[t]-hard for all t when the given graph is a tree [33])

Uniform Register Allocation

Instance: A structured program P; a positive integer k.

Question: Can P be compiled in such a way that all dynamically allocated variables are assigned to at most k general-purpose hardware registers without spilling?

Parameter: k

FPT (solvable in time $O(f(k) \cdot |P|)$ [22])

Note: A *structured program* is a program encoded in a high-level language without *goto*s. *Spilling* is the act of temporarily saving some variables on the stack in order to free the corresponding hardware registers. Compilers may reorder the program statements, as long as the data dependencies are not violated. See also Non-Uniform Register Allocation.

Unique Hitting Set

Instance: A set X; k subsets $X_1, \ldots, X_k$ of X.

Question: Is there a set $S \subseteq X$ such that for all i, $1 \leq i \leq k$, $|S \cap X_i| = 1$?

Parameter: k

FPT (by the reduction to problem kernel method [81, Exercise 3.2.5])

Unique Weighted CNF Satisfiability

Instance: A boolean formula X in conjunctive normal form; a positive integer k.

Question: Is there a unique weight k satisfying assignment for X?

Parameter: k

W[1]-hard, in W[2] (membership: direct proof ? [78]; hardness: reduction from Perfect Code [78])

Note: This problem belongs to $D_p[2] = \text{W}[2] \cap \text{co-W}[2]$ [77][81, Appendix A].

Unit Cycle Avoidance

Instance: A graph G; an abelian group of the form $A = (Z_2)^m$.

Question: Can the edges of G be labeled by elements of A so that no cycle in G has label sum equal to the identity element of A?

Parameter: A

nonuniform FPT (application of the Robertson-Seymour Theorem [193])

Note: Also in nonuniform FPT for an arbitrary abelian group A provided that G has maximum degree 3.

Unitary Quantified Boolean t-Normalized Formula Satisfiability

Instance: An integer r; a sequence $s_1, \ldots, s_r$ of pairwise disjoint sets of boolean variables; a boolean formula X involving the variables $s_1 \cup \ldots s_r$ that consists of t alternating layers of conjunctions and disjunctions with negations applied only to variables (t is a fixed constant).

Question: Is it the case that there exists a variable t_1 of s_1 such that for every variable t_2 of s_2, there exists a variable t_3 of s_3 such that ... (alternating qualifiers) such that, when the variables $t_1 \cup \ldots t_r$ are made true and all other variables are made false, formula X is true?

Parameter: r

AW[$*$]-complete (equivalent to Quantified Boolean t-Normalized Formula Satisfiability [1, 2, 81])

Vapnik-Chervonenkis Dimension

Instance: A family of subsets F of a base set X; a positive integer k.

Question: Is the Vapnik-Chervonenkis dimension of F at least k? The Vapnik-Chervonenkis dimension of a family of subsets F of a base set X is the maximum cardinality of a set $S \subseteq X$ such that for each subset $S' \subseteq S$, there exists $Y \in F$ such that $S \cap Y = S'$.

Parameter: k

W[1]-complete (membership: direct proof [79]; hardness: reduction from Clique [74])

Note: The general version of this problem is LOGNP-complete.

Vertex Average Min Linear Arrangement

Instance: A connected graph $G = (V, E)$; a positive integer k.

Question: Is there a one-to-one mapping $\sigma : V \to \{1, \ldots, |V|\}$ such that $\sum_{(u,v)\in E} |\sigma(u) - \sigma(v)| \leq k \cdot |V|$?

Parameter: k

para-NP-complete (the problem was introduced by Serna and Thilikos [198]; it is NP-complete for any $k \geq 2$ and is also para-NP-complete [140])

Note: The complexity class para-NP includes all parameterized problems with instances (I, k) that can be solved in time $O(f(k)\,|I|^c)$ by nondeterministic Turing machines [122, 124]. See also Linear Arrangement and Edge Average Min Linear Arrangement.

Vertex Average Profile

Instance: A connected graph $G = (V, E)$; a positive integer k.

Question: Is there a one-to-one mapping $\sigma : V \to \{1, \ldots, |V|\}$ (a linear arrangement) such that its profile is $\leq k \cdot |V|$?

Parameter: k

para-NP-complete (the problem was introduced by Serna and Thilikos [198]; it is NP-complete for any $k \geq 2$ and is also para-NP-complete [140])

Note: The *profile* of a linear arrangement is

$$\sum_{v\in V} (\sigma(v) - \min\{\sigma(w) : w \in N[v]\}).$$

The complexity class para-NP includes all parameterized problems with instances (I, k) that can be solved in time $O(f(k)\,|I|^c)$ by nondeterministic Turing machines [122, 124]. See also Linear Arrangement and Vertex Average Min Linear Arrangement.

Vertex Cover

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Does G have a vertex cover of size at most k? A *vertex cover* is a subset $V' \subseteq V$ such that $\forall\,(v, w) \in E$, $v \in V'$ or $w \in V'$.

Parameter: k

FPT (from a $O\big(k\,|V(G)| + 2^k k^{2k+2}\big)$ algorithm [41]; there is an impressive list of improvements on the klam value for this problem, see for example [75, 79, 179, 17, 87, 170, 202, 61, 171, 201, 62]; best current algorithms have time complexity in $O\big(r^k + |V(G)|\big)$ for $r \approx 1.27$, with trade-offs between small differences in r and leading constants, and many are also parallelizable [57]; it does not have a $O^*\Big(2^{o^{\text{eff}}(k)}\Big)$ algorithm unless M[1] = FPT, that is, unless q-CNF $\in$ DTIME $\Big(2^{o^{\text{eff}}(n)} \cdot \text{poly}(m)\Big)$ for all $q \geq 1$, where n is the number of variables and m is the size of the CNF formula [49, 73])

Parameter: the number of vertices of degree ≥ 3

FPT (by an exhaustive search tree $O\big(2^{\ell}\,|G|\big)$ algorithm that tries all possibilities for the ℓ vertices of degree ≥ 3 [137])

Parameter: treewidth of G

FPT ($O\big(2^{\text{tw}(G)}\,|V|\big)$ dynamic programming algorithm in [118])

Note: See also Planar Vertex Cover, Integer Weighted Vertex Cover, Real Weighted Vertex Cover, Log Vertex Cover, and Miniaturized Vertex Cover.

t-Vertex Cover

Instance: A graph $G = (V, E)$; positive integers k and t.

Question: Is there a t-vertex cover $C \subseteq V$ with $|C| \leq k$? A *t-vertex cover* is a subset of vertices such that $|\{e \in E : e \text{ is adjacent to some vertex in } C\}| = |E| - |E(G[V \setminus C])| \geq t$.

Parameter: k, t

Open (reported in [118])

Vertex Induced Forest

Instance: A graph $G = (V, E)$; a positive integer k.

Question: Is there a subset $F \subseteq V$, $|F| \geq k$, such that $G[F]$ is a forest, i.e., $G[F]$ does not contain any cycle?

Parameter: k

W[1]-complete ([156, 118]; in FPT by a $O^*(4.7316^k)$ algorithm when restricted to planar graphs [127, 118])

Note: This problem is the parameterized dual of Feedback Vertex Set when taking the number of vertices as size function.

Weight Distribution

Instance: A binary $m \times n$ matrix H; a positive integer k.

Question: Is there a set of at most k columns of H that sum to the all-zero vector?

Parameter: k

W[1]-hard, in W[2] ([89])

Note: See also Maximum Likelihood Decoding.

Weighted Antimonotone q-CNF Satisfiability

Instance: A boolean expression X in conjunctive normal form where each clause includes exactly q negated variables ($q \geq 2$), and no other negation is used; a positive integer k.

Question: Does X have a satisfying truth assignment of weight k?

Parameter: k

W[1]-complete (membership is trivial; hardness: direct proof [78, 81])

Weighted Antimonotone t-Normalized Satisfiability

Instance: A t-normalized boolean expression X ($t \geq 3$, t odd) where all variables are negated, and no other negation is used; a positive integer k.

Question: Does X have a satisfying truth assignment of weight k?

Parameter: k

W[t]-complete (direct proofs [75, 77])

Note: A boolean expression is *t-normalized* if it is of the form product-of-sums-of-products ... of literals with t alternations.

Weighted Antimonotone Formula Satisfiability

Instance: A boolean antimonotone formula X (all variables are negated, and no other negation is used); a positive integer k.

Question: Does X have a satisfying assignment of Hamming weight k?

Parameter: k

W[SAT]-complete (membership is trivial; hardness: reduction from Weighted Formula Satisfiability [1, 2])

Weighted Binary Integer Programming

Instance: A binary matrix A; a binary vector $\mathbf{b}$; an integer k.

Question: Does $A \cdot \mathbf{x} \geq \mathbf{b}$ have a binary solution of weight k?

Parameter: k

W[2]-complete (hardness: reduction from Weighted Monotone t-Normalized Satisfiability with $t = 2$ [77])

Note: See also Weighted Exact Binary Integer Programming.

Weighted Circuit Satisfiability

Instance: A boolean circuit C; a positive integer k.

Question: Does C have a satisfying assignment of Hamming weight k?

Parameter: k

W[P]-complete (kernel problem of the class W[P] [1, 2])

Weighted q-CNF Satisfiability

Instance: A boolean expression X in conjunctive normal form (CNF) such that each clause has no more than q literals; a positive integer k.

Question: Does X have a satisfying truth assignment of weight k?

Parameter: k

W[1]-complete (membership is trivial; hardness by a reduction from Independent Set [78, 81])

WEIGHTED CNF SATISFIABILITY

Instance: A boolean expression X in conjunctive normal form (CNF), a positive integer k.

Question: Does X have a satisfying truth assignment of weight k?

Parameter: k

W[2]-complete (membership is trival; hardness by direct proof [75, 77, 81])

Note: See also the WEIGHTED t-NORMALIZED SATISFIABILITY problem.

WEIGHTED EDGE DOMINATING SET

Instance: A graph $G = (V, E)$; a weight function $\omega : E \to \mathbb{R}_{\geq 1}$; a positive integer k.

Question: Is there a subset $D \subseteq E$ with $\sum_{e \in D} \omega(e) \leq k$ such that for each $e \in E$, either $e \in D$ or there exists $e' \in D$ that is incident on e?

Parameter: k

FPT ($O^*\big(4^k\big)$ algorithm in [118])

Note: This is a generalization of EDGE DOMINATING SET.

WEIGHTED EXACT BINARY INTEGER PROGRAMMING

Instance: A binary matrix A; a binary vector $\mathbf{b}$; an integer k.

Question: Does $A \cdot \mathbf{x} = \mathbf{b}$ have a binary solution of weight k?

Parameter: k

W[1]-hard ([77])

Note: See also WEIGHTED BINARY INTEGER PROGRAMMING.

WEIGHTED EXACT CNF SATISFIABILITY

Instance: A boolean expression E in conjunctive normal form; a positive integer k.

Question: Is there a truth-assignment of weight k to the variables of E that makes exactly one literal in each clause of E true?

Parameter: k

W[1]-complete (equivalent to Perfect Code [78])

Note: See also Unique Weighted CNF Satisfiability.

Weighted Formula Satisfiability

Instance: A boolean formula X; a positive integer k.

Question: Does X have a satisfying assignment of Hamming weight k?

Parameter: k

W[SAT]-complete (kernel problem of the class W[SAT] [1, 2])

Note: This problem is also known as Weighted Satisfiability.

Weighted Monotone Formula Satisfiability

Instance: A boolean monotone formula X; a positive integer k.

Question: Does X have a satisfying assignment of Hamming weight k?

Parameter: k

W[SAT]-complete (membership is trivial; hardness: reduction from Weighted Formula Satisfiability [1, 2])

Weighted Monotone Circuit Satisfiability

Instance: A boolean monotone circuit C; a positive integer k.

Question: Does C have a satisfying assignment of Hamming weight k?

Parameter: k

W[P]-complete (membership is trivial; hardness: reduction from Minimum Axiom Set [1, 2])

Weighted Monotone t-Normalized Satisfiability

Instance: A t-normalized boolean expression X ($t \geq 2$, t even) without negations; a positive integer k.

Question: Does X have a satisfying truth assignment of weight k?

Parameter: k

W[t]-complete (direct proofs [75, 77])

Note: A boolean expression is *t-normalized* if it is of the form product-of-sums-of-products ... of literals with t alternations. Since a 2-normalized boolean expression is in conjunctive normal form, for $t = 2$ the problem coincides with Weighted CNF Satisfiability.

Weighted t-Normalized Satisfiability

Instance: A t-normalized boolean expression X ($t \geq 2$); a positive integer k.

Question: Does X have a satisfying truth assignment of weight k?

Parameter: k

W[t]-complete (direct proofs [75, 77, 81])

Note: A boolean expression is *t-normalized* if it is of the form product-of-sums-of-products ... of literals with t alternations. Since a 2-normalized boolean expression is in conjunctive normal form, for $t = 2$ the problem coincides with Weighted CNF Satisfiability.

Weighted t-Normalized W[*] Satisfiability

Instance: A boolean expression X that is an n-product of k-sums of n-products of k-sums ... (repeated so that the number of product terms is t, $t \geq 2$).

Question: Does X have a satisfying truth assignment of weight k?

Parameter: k

W[t]-complete (direct proofs [76]; the argument for completeness proceeds from first principles and follows the same style as the basic proofs for W[t]-completeness of Weighted t-Normalized Satisfiability)

Weighted Planar Circuit Satisfiability

Instance: A planar decision circuit C; a positive integer k.

Question: Does C have a satisfying assignment of Hamming weight k?

Parameter: k

W[P]-complete (membership is trivial; hardness: reduction from Weighted Circuit Satisfiability [2])

Alphabetical index

Hierarchical index

FPT

nonuniform FPT

randomized FPT

M[1]-complete

W[1]-complete

W[1]-hard, in **W**[2]

W[1]-hard, in **W**[**P**]

W[1]-hard

W[2]-complete

W[2]-hard, in W[P]

W[2]-hard

W[t]-complete

W[t]-hard

W[t]-hard for all t, in W[P]

W[t]-hard for all t

M[SAT]-complete

W[SAT]-complete

W[SAT]-hard

M[P]-complete

W[P]-complete

W[P]-hard

AW[$*$]-complete

AW[SAT]-complete

AW[SAT]-hard

AW[P]-complete

AW[P]-hard

para-NP-complete

XP-complete

Open

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